Apportionment:

Identical items are to be distributed among some groups in a way that is proportional to their populations, and the items can't be broken into pieces.

Example:

Group	A	B	C	Total
Population	9	6	5	20

The three groups are to share 9 items proportionally. So Group A should get $\frac{9}{20}$ of the 9 items, Group B should get $\frac{6}{20}$ of the 9 items, and Group C should get $\frac{5}{20}$ of the 9 items. Unfortunately, this leads to Group A getting $4\frac{1}{20}$ items, Group B getting $2\frac{7}{10}$ items, and Group C getting $2\frac{1}{4}$ items. What do you do?

Terminology:

The Standard Divisor:

$$\text{Standard Divisor} = \frac{\text{total population}}{\text{number of items to be allocated}}$$

Example:

$$\text{Standard Divisor} = \frac{20}{9} = 2.\overline{2}$$

The Standard Quota:

$$\text{Standard Quota} = \frac{\text{population of a particular group}}{\text{standard divisor}}$$

Example:

Group	A	B	C	Total
Population	9	6	5	20
Standard Quota	$\frac{9}{2.\overline{2}} = 4.05$	$\frac{6}{2.\overline{2}} = 2.7$	$\frac{5}{2.\overline{2}} = 2.25$	9

Lower Quota:

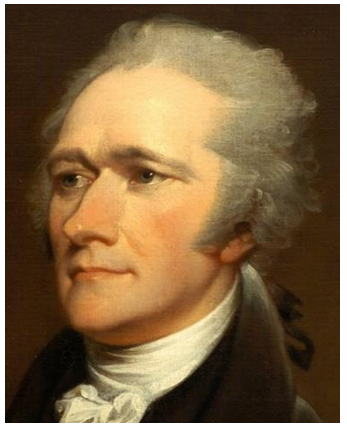
It's the standard quota rounded down to the nearest whole number.

Upper Quota:

It's the standard quota rounded up to the nearest whole number.

Example:

Group	A	B	C	Total
Population	9	6	5	20
Standard Quota	4.05	2.7	2.25	9
Lower Quota	4	2	2	8
Upper Quota	5	3	3	11



Hamilton's Apportionment Method:

1. Start by giving each group its lower quota.

2. Distribute the remaining items one at a time to the groups with the largest decimal parts in their standard quotas, until all the items have been allocated.

Example:

Group	A	B	C	Total
Population	9	6	5	20
Standard Quota	4.05	2.7	2.25	9
Lower Quota	4	2	2	8
Hamilton Apportionment	4	3	2	9

Another example:

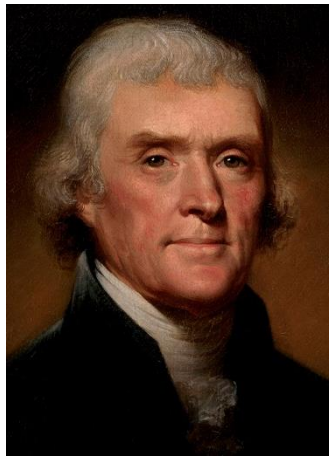
Group	A	B	C	D	Total
Population	138	266	534	662	1600

200 items are to be shared. Standard Divisor $= \frac{1600}{200} = 8$.

Group	A	B	C	D	Total
Population	138	266	534	662	1600
Standard Quota	17.25	33.25	66.75	82.75	200
Lower Quota	17	33	66	82	198
Hamilton Apportionment	17	33	67	83	200

Jefferson's Apportionment Method:

Find a modified divisor, *d*, usually smaller than the standard divisor, so that the sum of the lower modified quotas is equal to the number of items to be allocated. Apportion each group its lower modified quota.



Example:

Group	A	B	C	Total
Population	9	6	5	20

9 items are to be allocated. The standard divisor is $\frac{20}{9} = 2.\overline{2}$, so we'll try a modified divisor of $d = 1.9$.

Group	A	B	C	Total
Population	9	6	5	20
Modified Quota	$\frac{9}{1.9} = 4.736...$	$\frac{6}{1.9} = 3.157...$	$\frac{5}{1.9} = 2.631...$	
Lower Modified Quota	4	3	2	9

This is the Jefferson Apportionment.

Another example:

Group	A	B	C	D	Total
Population	138	266	534	662	1600

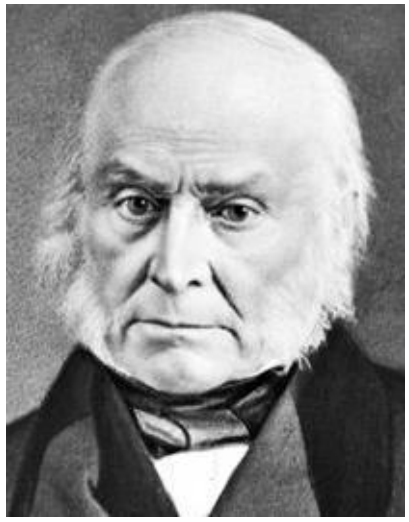
200 items are to be shared. Standard Divisor $= \frac{1600}{200} = 8$**, so we'll try a modified divisor of** $d = 7.9$**.**

Group	A	B	C	D	Total
Population	138	266	534	662	1600
Modified Quota	17.46…	33.67…	67.59…	83.79…	
Lower Modified Quota	17	33	67	83	200

This is the Jefferson Apportionment.

Adams's Apportionment Method:

Find a modified divisor, *d*, usually larger than the standard divisor, so that the sum of the upper modified quotas is equal to the number of items to be allocated. Apportion each group its upper modified quota.



Example:

Group	A	B	C	Total
Population	9	6	5	20

9 items are to be allocated. The standard divisor is $\frac{20}{9} = 2.\overline{2}$, so we'll try a modified divisor of $d = 2.6$.

Group	A	B	C	Total
Population	9	6	5	20
Modified Quota	$\frac{9}{2.6}=3.461...$	$\frac{6}{2.6}=2.307...$	$\frac{5}{2.6}=1.923...$	
Upper Modified Quota	4	3	2	9

This is the Adams Apportionment.

Another example:

Group	A	B	C	D	Total
Population	138	266	534	662	1600

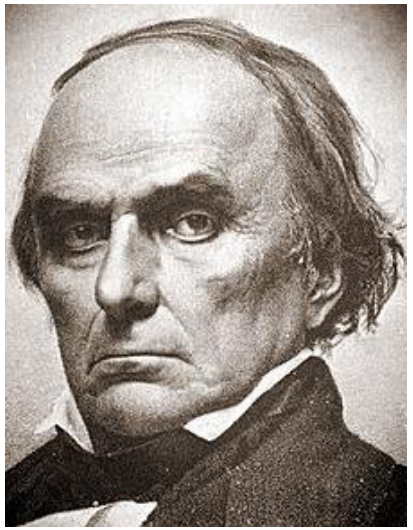
200 items are to be shared. Standard Divisor $=\frac{1600}{200}=8$**, so we'll try a modified divisor of** $d=8.08$**.**

Group	A	B	C	D	Total
Population	138	266	534	662	1600
Modified Quota	17.07…	32.92…	66.08…	81.93…	
Upper Modified Quota	18	33	67	82	200

This is the Adams Apportionment.

Webster's Apportionment Method:

Find a modified divisor, *d*, so that the sum of the rounded modified quotas is equal to the number of items to be allocated. Apportion each group its rounded modified quota.



Example:

Group	A	B	C	Total
Population	9	6	5	20

9 items are to be allocated. The standard divisor is $\frac{20}{9} = 2.\overline{2}$, so we'll try a modified divisor of $d = 2.1$.

Group	A	B	C	Total
Population	9	6	5	20
Modified Quota	$\frac{9}{2.1} = 4.285...$	$\frac{6}{2.1} = 2.857...$	$\frac{5}{2.1} = 2.38...$	
Rounded Modified Quota	4	3	2	9

This is the Webster Apportionment.

Another example:

Group	A	B	C	D	Total
Population	138	266	534	662	1600

200 items are to be shared. Standard Divisor $= \frac{1600}{200} = 8$**, so we'll try a modified divisor of** $d = 8$**.**

Group	A	B	C	D	Total
Population	138	266	534	662	1600
Modified Quota	17.25	33.25	66.75	82.75	
Rounded Modified Quota	17	33	67	83	200

This is the Webster Apportionment.

All of the apportionment methods that we've discussed can fail. A simple example would be if all the states have the same population, the number of states is even, and there is an odd number of items to be allocated.

Apportionment Summary:

Hamilton's Method either gives a state/group its lower or upper quota. Its use was vetoed by George Washington following the 1790 census in favor of Jefferson's Method. It's theorized that he did this because Jefferson's Method gave Washington's large home state of Virginia one more seat than Hamilton's Method. It was used from 1852 to 1910.

Jefferson's Method tends to reward larger states/groups over smaller ones, and can result in a state/group receiving no items. States/groups can receive less than their lower quota or more than their upper quota. It was used from 1792 to 1842.

Adams' Method tends to reward smaller states/groups over larger ones, and it will always give each state/group at least 1 item. States/groups can receive less than their lower quota or more than their upper quota. It was never used.

Webster's Method tends to reward larger states/groups over smaller ones less than Jefferson's Method. It's more likely that a state will receive its lower or upper quota with Webster's Method than with Jefferson's Method. States/groups can receive less than their lower quota or more than their upper quota. It was used from 1842 to 1852 and again from 1910 to 1940.