Traveling Salesman Problem:

It's the problem of finding a Hamilton circuit in a complete weighted graph that has the smallest weight. Such a Hamilton circuit is called an optimal solution of the problem.

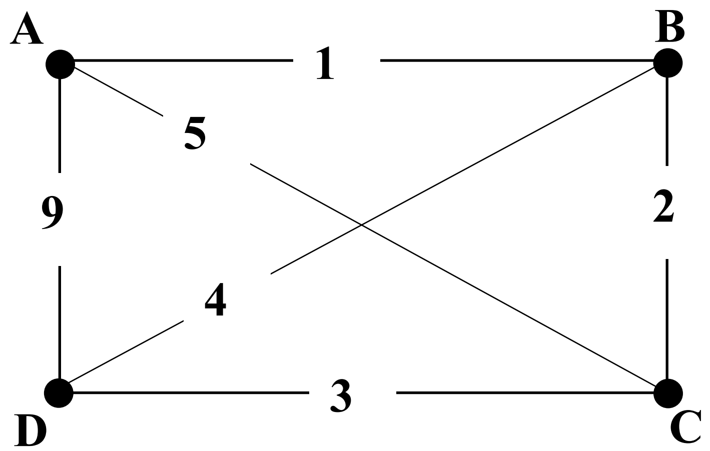
Brute Force Method:

1. List all the Hamilton circuits.

2. Determine the weight of each of the listed Hamilton circuits.

3. Hamilton circuits with the minimum weight are optimal solutions.

Examples:



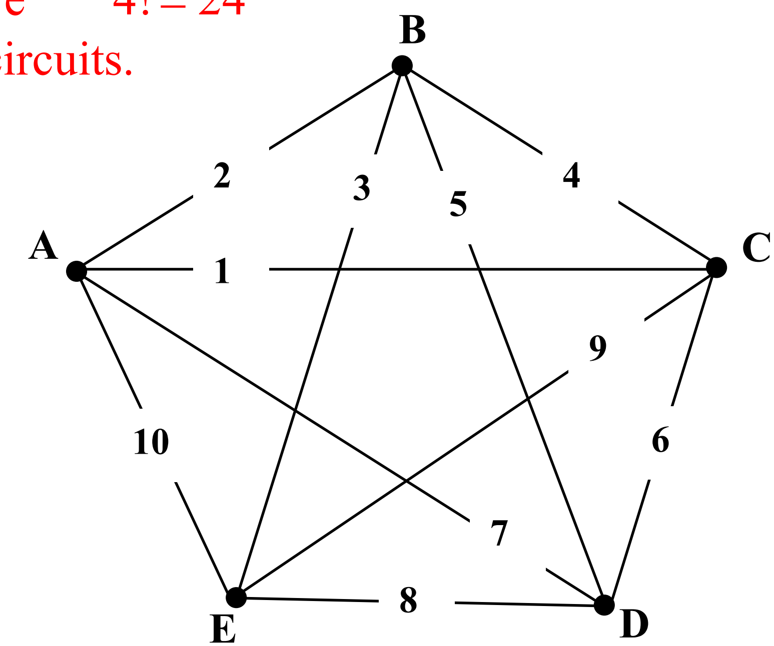
There are 4 vertices in this complete graph, so there are $3! = 6$ Hamilton circuits.

	Hamilton circuit	Weight
	A,B,C,D,A	15
	A,B,D,C,A	13
	A,C,B,D,A	20
reversal of A,B,D,C,A	A,C,D,B,A	13
reversal of A,C,B,D,A	A,D,B,C,A	20
reversal of A,B,C,D,A	A,D,C,B,A	15

The weight values occur in pairs, since each circuit and its reversal occur in the list. This can be used to efficiently fill in the weight values in the table.

The optimal solutions are A,B,D,C,A and its reversal A,C,D,B,A.

There are 5 vertices in this complete graph, so there are $4! = 24$ Hamilton circuits.



Hamilton Circuit	Weight	
A,B,C,D,E,A	30	
A,B,C,E,D,A	30	
A,B,D,C,E,A	32	
A,B,D,E,C,A	25	
A,B,E,C,D,A	27	
A,B,E,D,C,A	20	
A,C,B,D,E,A	28	
A,C,B,E,D,A	23	
A,C,D,B,E,A	25	
A,C,D,E,B,A	20	
A,C,E,B,D,A	25	
A,C,E,D,B,A	25	
A,D,B,C,E,A	35	
A,D,B,E,C,A	25	
A,D,C,B,E,A	30	
A,D,C,E,B,A	27	
A,D,E,B,C,A	23	
A,D,E,C,B,A	30	reversal of A,B,C,E,D,A
A,E,B,C,D,A	30	reversal of A,D,C,B,E,A
A,E,B,D,C,A	25	reversal of A,C,D,B,E,A
A,E,C,B,D,A	35	reversal of A,D,B,C,E,A
A,E,C,D,B,A	32	reversal of A,B,D,C,E,A
A,E,D,B,C,A	28	reversal of A,C,B,D,E,A
A,E,D,C,B,A	30	reversal of A,B,C,D,E,A

The optimal solutions are A,B,E,D,C,A and its reversal A,C,D,E,B,A.

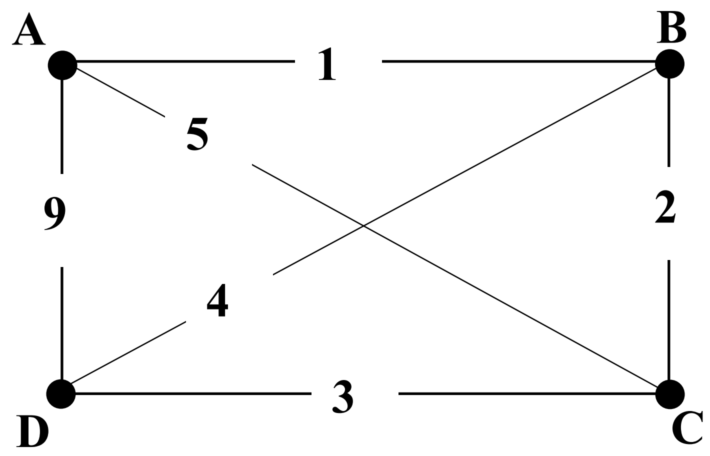
There is a method for producing approximate optimal solutions called the *<u>Nearest Neighbor Method.</u>*

1. Choose a starting vertex.

2. Choose the edge with the smallest weight.

3. From the next vertex, choose the edge with the smallest weight that doesn't lead to a previously visited vertex.

4. Continue until all vertices are visited exactly once, and you've returned to the starting vertex.

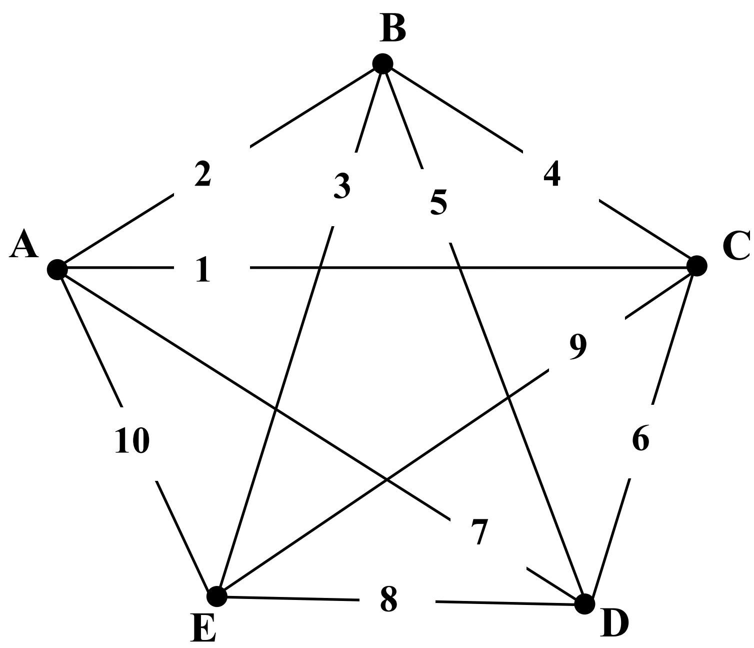


Find a Nearest Neighbor approximate optimal solution starting at the given vertex.

Starting vertex	**Nearest Neighbor solution**
A	A,B,C,D,A *(with a weight of 15)*
B	B,A,C,D,B *(with a weight of 13)*

The Nearest Neighbor solution B,A,C,D,B is the same as the optimal solution(s) we found earlier of A,B,D,C,A and its reversal A,C,D,B,A.

Notice that the Nearest Neighbor solution starting with vertex A did not find an optimal solution!



Find a Nearest Neighbor approximate optimal solution starting at the given vertex.

Starting vertex	Nearest Neighbor solution
A	A,C,B,E,D,A *(with a weight of 23)*
D	D,B,A,C,E,D *(with a weight of 25)*

Neither of these Nearest Neighbor solutions is the same as the optimal solution(s) that we found earlier of A,B,E,D,C,A and its reversal A,C,D,E,B,A.

If there is tie between/among two or more edge weights, then the Nearest Neighbor Method won't produce a unique Hamilton circuit. Produce each one of the circuits and choose the circuit(s) with the smallest weight.