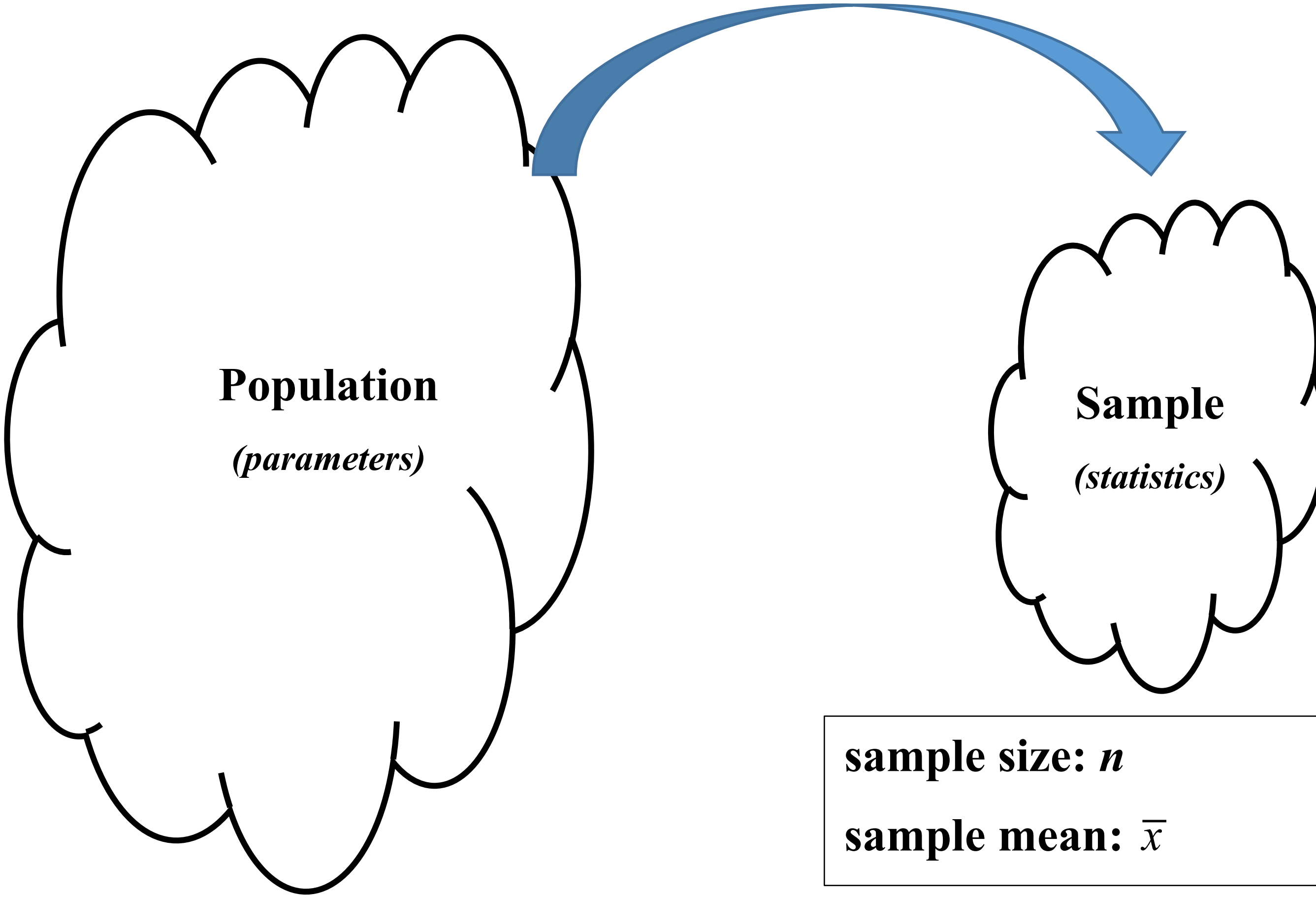
## ***Making Predictions about the Population Mean from the Sample Mean :***




**sample size:** $n$

**sample mean:** $\bar{x}$

**population mean:** $\mu$

**population standard deviation:** $\sigma$

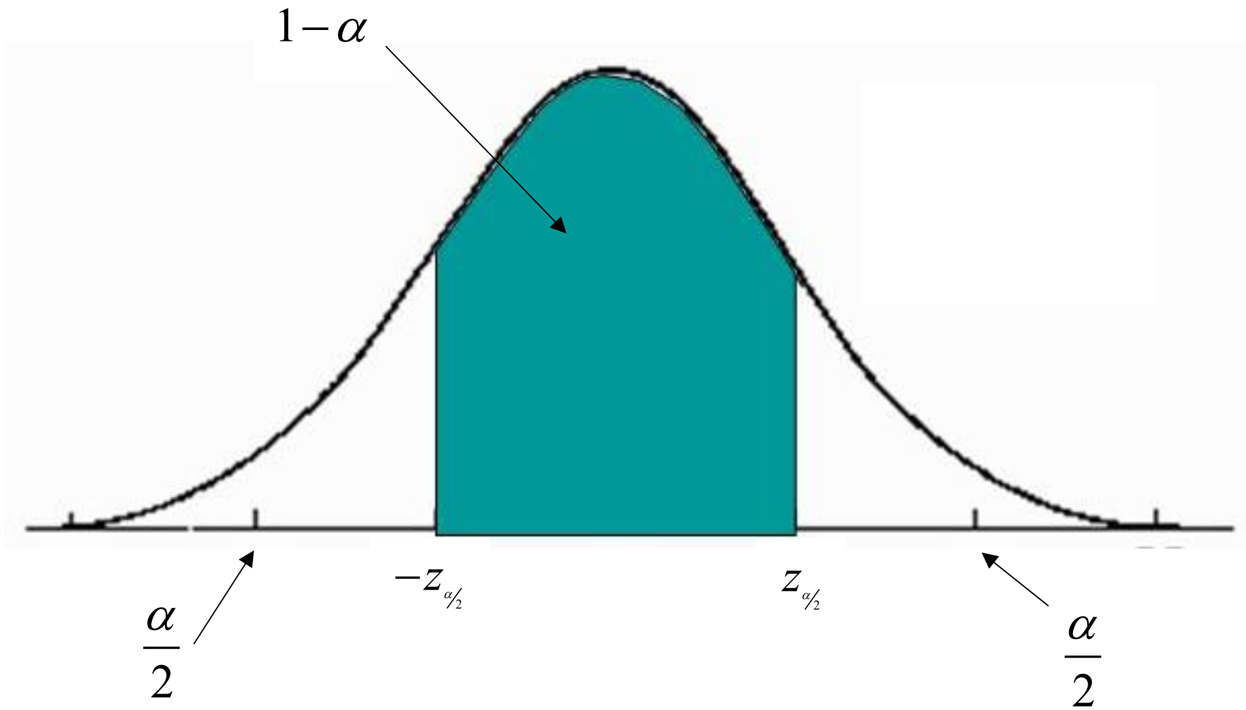
**We can definitely use the sample mean as an estimate for the population mean, but how accurate will it be?**

**If the sample size is large($n > 30$) or the population is normal, then the sample mean, $\bar{x}$, is normal(approximately) with mean equal to the population mean and standard deviation of $\frac{\sigma}{\sqrt{n}}$, called the *Standard Error of the Mean*.**

**We want to come up with an interval centered at the sample mean, $\bar{x}$, that contains the population mean, $\mu$, with a certain probability. This interval is called a confidence interval for the population mean.**

**The confidence interval can be written as $\overline{x} \pm E$ or $(\overline{x} - E, \overline{x} + E)$, and the probability that it contains the population mean is called its confidence level, and it's usually written as a $(1-\alpha)$ confidence interval. The width of the confidence interval is $2 \cdot E$.**

**So we want $P(\overline{x} - E < \mu < \overline{x} + E) = 1 - \alpha$, and this is equivalent to $P(\mu - E < \overline{x} < \mu + E) = 1 - \alpha$. Now let's subtract the mean and divide by the standard deviation.** $P\left( -\frac{E}{\frac{\sigma}{\sqrt{n}}} < \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}} < \frac{E}{\frac{\sigma}{\sqrt{n}}} \right) = 1 - \alpha$ **or just** $P\left( -\frac{E}{\frac{\sigma}{\sqrt{n}}} < Z < \frac{E}{\frac{\sigma}{\sqrt{n}}} \right) = 1 - \alpha$**.**




$z_{\alpha/2}$ **is called a critical value so that** $P\left(Z > z_{\alpha/2}\right) = \frac{\alpha}{2}$**, and therefore that** $P\left(-z_{\alpha/2} < Z < z_{\alpha/2}\right) = 1-\alpha$**.**

**So for our confidence interval to work, it must be that** $\frac{E}{\frac{\sigma}{\sqrt{n}}} = z_{\alpha/2}$**, or** $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$**. So the** $(1-\alpha)$ **confidence interval can be written as** $\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ **or** $\left( \bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right)$**.**

**For a 90% confidence interval,** $1-\alpha = .9$**, so** $\alpha = .1$**, and** $\frac{\alpha}{2} = .05$**.** $z_{.05} = 1.645$**.**

**For a 95% confidence interval,** $1-\alpha = .95$**, so** $\alpha = .05$**, and** $\frac{\alpha}{2} = .025$**.** $z_{.025} = 1.96$**.**

**For a 99% confidence interval,** $1-\alpha = .99$**, so** $\alpha = .01$**, and** $\frac{\alpha}{2} = .005$**.** $z_{.005} = 2.576$**.**

**Examples:**

1. **A sample of size 16 is randomly selected from a normal population with a standard deviation of 5.23. If the sample mean is 15.1, then construct confidence intervals with the following confidence levels.**

   **a) 90%**

$$15.1 \pm 1.645 \cdot \frac{5.23}{\sqrt{16}}$$

$$15.1 \pm 2.158837...$$

$$\boxed{15.1 \pm 2.16}\,(\text{round to one more place than the mean})$$

$$\text{or } \boxed{(12.94, 17.26)}$$

**width of the confidence interval:** 4.32

   **b) 95%**

$$15.1 \pm 1.96 \cdot \frac{5.23}{\sqrt{16}}$$

$$\boxed{15.1 \pm 2.56}$$

$$\text{or } \boxed{(12.54, 17.66)}$$

**width of the confidence interval:** 5.12

**c) 99%**

$$15.1 \pm 2.576 \cdot \frac{5.23}{\sqrt{16}}$$

$$\boxed{15.1 \pm 3.37}$$

$$\text{or } \boxed{(11.73, 18.47)}$$

**width of the confidence interval:** 6.74

**2. A sample of size 36 is randomly selected from a population with a standard deviation of 4.56. If the sample mean is 20.4, then construct confidence intervals with the following confidence levels.**

**a) 90%**

$$20.4 \pm 1.645 \cdot \frac{4.56}{\sqrt{36}}$$

$$\boxed{20.4 \pm 1.25}$$

**width of the confidence interval:** 2.5

**b) 95%**

$$20.4 \pm 1.96 \cdot \frac{4.56}{\sqrt{36}}$$

$$\boxed{20.4 \pm 1.49}$$

**width of the confidence interval:** 2.98

**c) 99%**

$$20.4 \pm 2.576 \cdot \frac{4.56}{\sqrt{36}}$$

$$\boxed{20.4 \pm 1.96}$$

**width of the confidence interval:** 3.92

***Finding the Sample Size for a Desired Level of Accuracy:***

**Suppose that you would like to know how large a sample is required to estimate a population mean with an error of at most *E* at a** $(1-\alpha)$ **confidence.**

$$z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le E$$

$$\frac{1}{\sqrt{n}} \le \frac{E}{z_{\alpha/2} \cdot \sigma}$$

$$\sqrt{n} \ge \frac{z_{\alpha/2} \cdot \sigma}{E}$$

$$n \ge \left( \frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

**Examples:**

1. **You would like to estimate the population mean of a population with a standard deviation of 1.2 with an error of at most .05 with 90% confidence. How large of a sample should you take?**

$$n \geq \left( \frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2 = \left( \frac{1.645 \cdot 1.2}{.05} \right)^2 = 1558.6704$$

***n* has to be a whole number, so the sample size must be 1559 or larger.**

2. **You would like to estimate the population mean of a population with a standard deviation of 1.8 with an error of at most .08 with 95% confidence. How large of a sample should you take?**

$$n \geq \left( \frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2 = \left( \frac{1.96 \cdot 1.8}{.08} \right)^2 = 1944.81$$

***n* has to be a whole number, so the sample size must be 1945 or larger.**