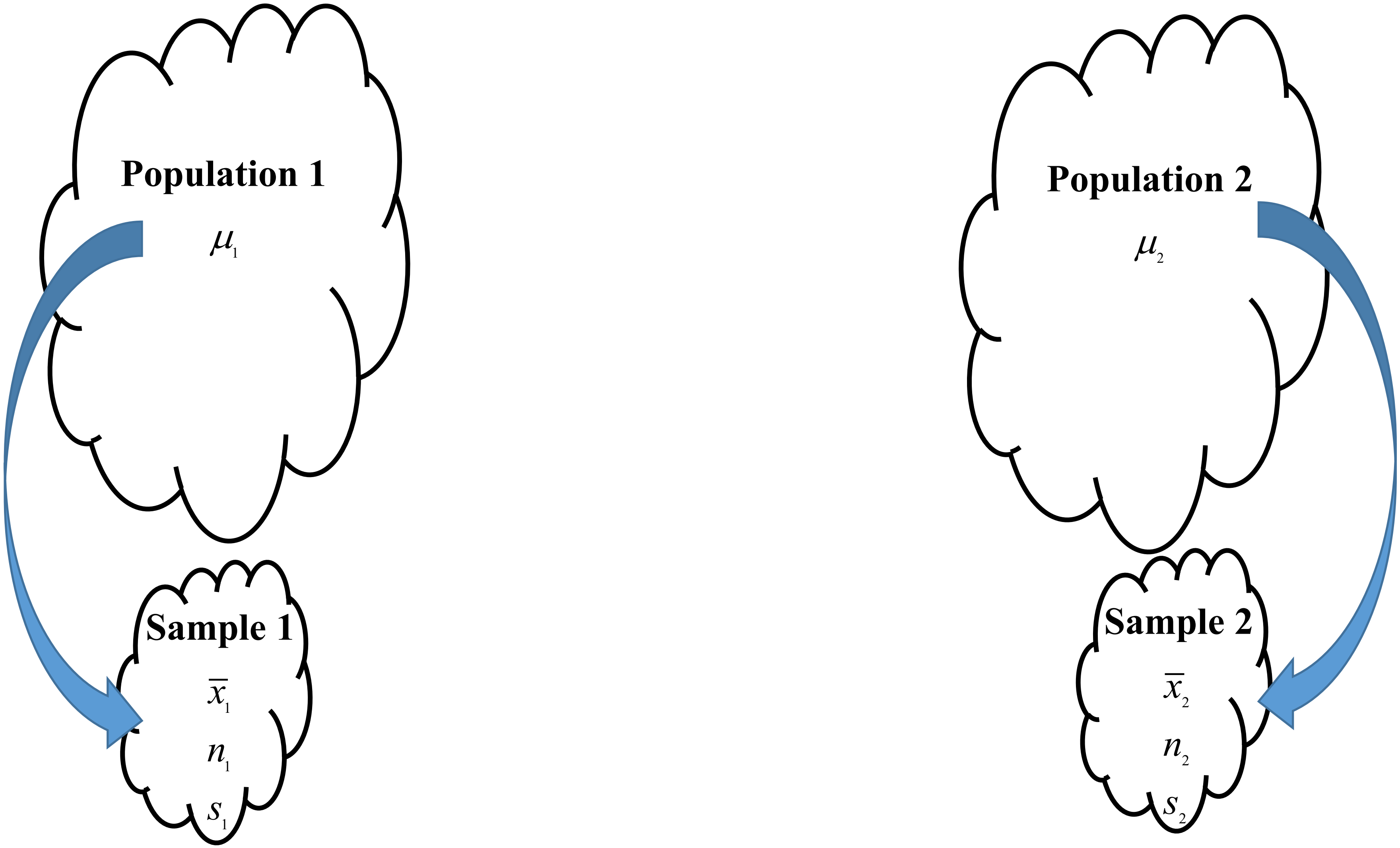
Hypothesis Testing for Two Population Means(Independent Samples):



Assumptions: $n_1, n_2 > 30$**, or the populations are normal.**

The null hypothesis states that the two population means are equal, i.e. $\mu_1 = \mu_2$**. The three possible hypothesis tests involving the two population means are**

$$\begin{matrix} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 < \mu_2 \end{matrix}$$ **, called a left-tail test**

$$\begin{matrix} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 > \mu_2 \end{matrix}$$ **, called a right-tail test**

$$\begin{matrix} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 \neq \mu_2 \end{matrix}$$ **, called a two-tail test**

Independent samples from both populations are taken, and if, under the assumption of the null hypothesis, the test statistic is very rare, then the null hypothesis is rejected in favor of the alternative hypothesis. If the test statistic is not very rare, then the null hypothesis is not rejected.

The criterion of what is rare is set before the hypothesis test is performed. It's called the level of significance of the hypothesis test and is abbreviated as α.

Examples:

1. Perform the (left-tail)hypothesis test

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 < \mu_2$$

at the level of significance, α.

We would take our samples, if the value of the test statistic is less than or equal to $-t_\alpha$, then we'd reject H_0.

2. Perform the (right-tail)hypothesis test

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 > \mu_2$$

at the level of significance, α.

We would take our samples, if the value of the test statistic is greater than or equal to t_α, then we'd reject H_0.

3. Perform the (two-tail)hypothesis test

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

at the level of significance, α.

We would take our samples, if the value of the test statistic is less than or equal to $-t_{\alpha/2}$ or greater than or equal to $t_{\alpha/2}$, then we'd reject H_0.

The test statistic $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ has Student-t distribution with degrees of freedom equal to either the complicated formula, $\dfrac{\left[\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right]^2}{\dfrac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \dfrac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$ rounded down to the nearest whole number or the simple formula of the minimum of $(n_1 - 1)$ and $(n_2 - 1)$.

The complicated formula is more accurate and is used by most textbooks and software packages.

Examples:

1. Perform the following hypothesis test at the $\alpha = .05$ level of significance.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 < \mu_2$$

With $\bar{x}_1 = 6.5, s_1 = 1.12, n_1 = 36$ **and** $\bar{x}_2 = 7, s_2 = 1.35, n_2 = 36$

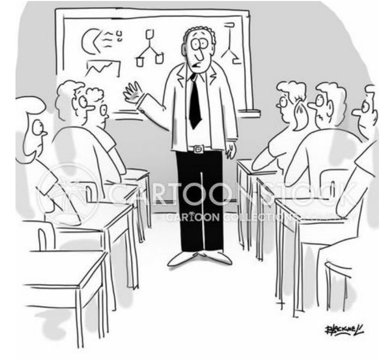
$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{6.5 - 7}{\sqrt{\frac{(1.12)^2}{36} + \frac{(1.35)^2}{36}}} = -1.710269...$$

Easy degrees of freedom: minimum of $(36-1)$ **and** $(36-1)$ **is 35.**

Is $-1.710269... \leq -1.69$**?**

Complicated degrees of freedom: $\dfrac{\left[\dfrac{(1.12)^2}{36}+\dfrac{(1.35)^2}{36}\right]^2}{\dfrac{\left(\frac{1.12^2}{36}\right)^2}{36-1}+\dfrac{\left(\frac{1.35^2}{36}\right)^2}{36-1}} = 67.69238871\ldots$**, which rounds down to 67.**

Is $-1.710269\ldots \leq -1.671$**?**



"I'M SORRY... BUT I'VE JUST BEEN INFORMED RECENT TECHNOLOGICAL ADVANCES HAVE RENDERED THIS COURSE OBSOLETE!"

2. Perform the following hypothesis test at the $\alpha = .05$ level of significance.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 > \mu_2$$

With $\bar{x}_1 = 8, s_1 = 2.13, n_1 = 81$ **and** $\bar{x}_2 = 7.5, s_2 = 1.96, n_2 = 81$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{8 - 7.5}{\sqrt{\frac{(2.13)^2}{81} + \frac{(1.96)^2}{81}}} = 1.554638...$$

Easy degrees of freedom: minimum of $(81-1)$ **and** $(81-1)$ **is 80.**

Is $1.554638... \geq 1.664$**?**

3. Perform the following hypothesis test at the $\alpha = .05$ level of significance.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

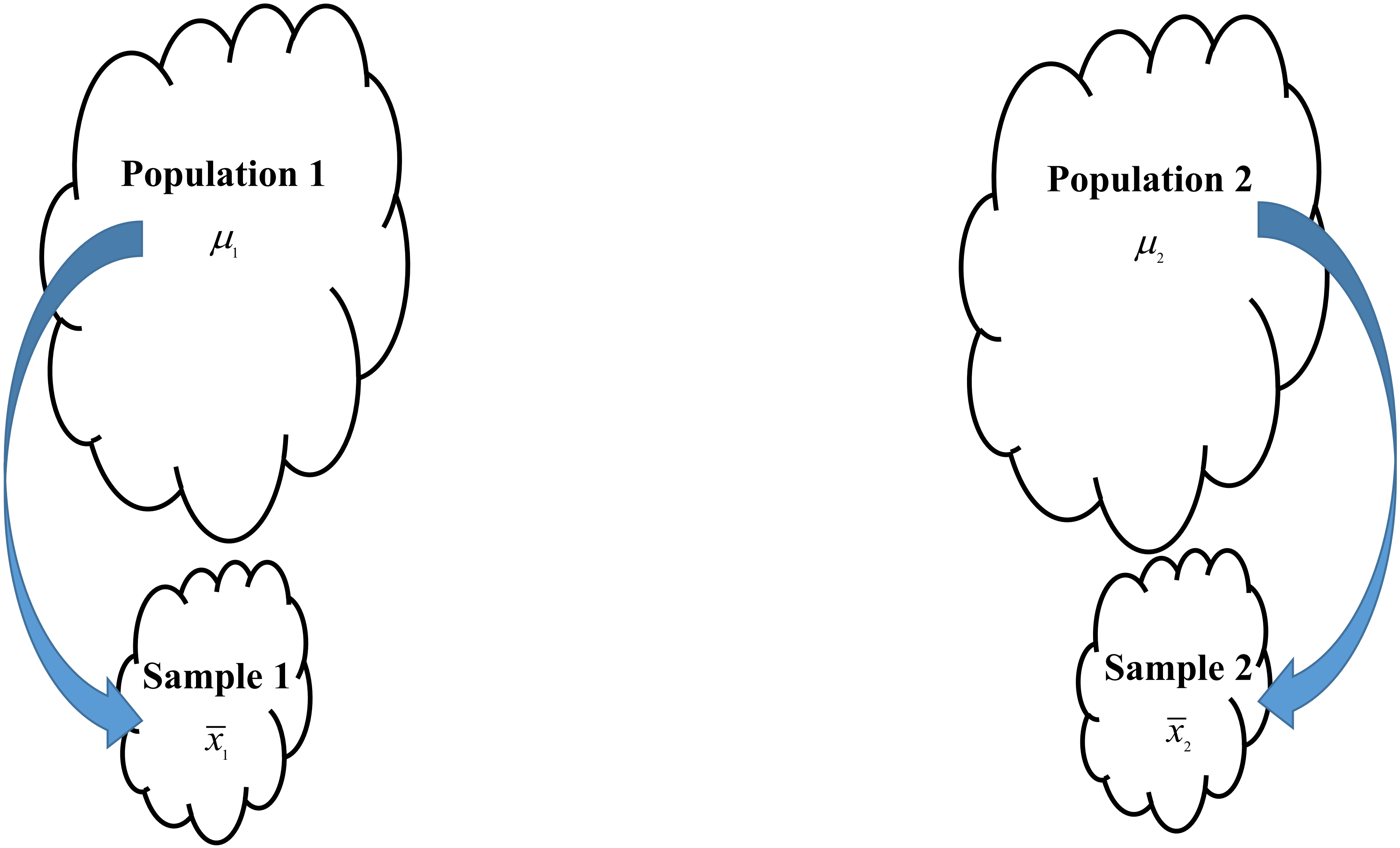
With $\overline{x}_1 = 9, s_1 = 1.35, n_1 = 36$ **and** $\overline{x}_2 = 10, s_2 = 1.72, n_2 = 40$

$$t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{9 - 10}{\sqrt{\frac{(1.35)^2}{36} + \frac{(1.72)^2}{40}}} = -2.833134...$$

Easy degrees of freedom: minimum of $(36-1)$ **and** $(40-1)$ **is 35.**

Is $-2.833134... \leq -2.03$**?**

Hypothesis Testing for Two Population Means(Dependent Samples):



Now the populations and samples are dependent on each other. Population 1 might be people before receiving a medication while Population 2 is the same people after receiving the medication. Population 1 is wives, and Population 2 is their husbands. When a sample is randomly selected from one of the populations, it automatically determines the sample from the other population. The samples are referred to as paired samples.

The difference in the values between Population 1 and Population 2 is abbreviated as *d*, where $d = \text{Population 1 value} - \text{Population 2 value}$**.** $\bar{d}$ **is the sample mean of the paired differences.** s_d **is the sample standard deviation of the paired differences.** μ_d **is the population mean of the differences.**

$$\mu_d = \mu_1 - \mu_2 \text{ and } \bar{d} = \bar{x}_1 - \bar{x}_2$$

The null hypothesis states that the population mean of the differences is zero, i.e. $\mu_d = 0 \left(\mu_1 = \mu_2 \right)$. The three possible hypothesis tests involving the population mean of the differences are

$$\begin{aligned} H_0 &: \mu_d = 0 \\ H_1 &: \mu_d < 0 \end{aligned}$$, **called a left-tail test**

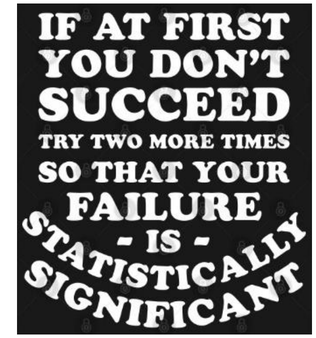
$$\begin{aligned} H_0 &: \mu_d = 0 \\ H_1 &: \mu_d > 0 \end{aligned}$$, **called a right-tail test**

$$\begin{aligned} H_0 &: \mu_d = 0 \\ H_1 &: \mu_d \neq 0 \end{aligned}$$, **called a two-tail test**

Dependent samples from both populations are taken, and if, under the assumption of the null hypothesis, the test statistic is very rare, then the null hypothesis is rejected in favor of the alternative hypothesis. If the test statistic is not very rare, then the null hypothesis is not rejected.

The criterion of what is rare is set before the hypothesis test is performed. It's called the level of significance of the hypothesis test and is abbreviated as α.

Assuming that $n > 30$ or the differences are approximately normal, the test statistic $t = \dfrac{\bar{d}}{\frac{s_d}{\sqrt{n}}}$ has Student-t distribution with degrees of freedom equal to $n-1$.



Examples:

1. Perform the following hypothesis test at the $\alpha = .10$ level of significance.

$$H_0 : \mu_d = 0$$

$$H_1 : \mu_d > 0$$

Sample 1	**19**	**15**	**16**	**23**	**24**
Sample 2	**18**	**19**	**10**	**14**	**17**
Difference	**1**	**-4**	**6**	**9**	**7**

$$\bar{d} = \frac{1 + -4 + 6 + 9 + 7}{5} = \frac{19}{5} = 3.8$$

$$s_d = \sqrt{\frac{(1-3.8)^2 + (-4-3.8)^2 + (6-3.8)^2 + (9-3.8)^2 + (7-3.8)^2}{4}} = \sqrt{\frac{110.8}{4}} = 5.26$$

$$t = \frac{3.8}{\frac{5.26}{\sqrt{5}}} = 1.61541...$$

Is $1.61541... \geq 1.533$**?**

2. Perform the following hypothesis test at the $\alpha = .05$ level of significance.

$$H_0 : \mu_d = 0$$
$$H_1 : \mu_d < 0$$

Sample 1	5	1	5	5	5	2	2
Sample 2	**6**	**3**	**4**	**5**	**7**	**1**	**4**
Difference	**-1**	**-2**	**1**	**0**	**-2**	**1**	**-2**

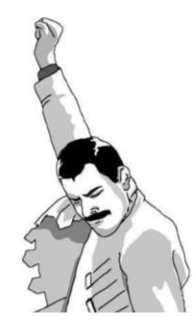
$$\bar{d} = \frac{-1+-2+1+0+-2+1+-2}{7} = \frac{-5}{7} = -.71428\ldots \approx -.7$$

$$s_d = \sqrt{\frac{11.43}{6}} = 1.38$$

$$t = \frac{-.7}{\frac{1.38}{\sqrt{7}}} = -1.34204\ldots$$

Is $-1.34204\ldots \leq -1.943$**?**

$p<0.05$



3. Perform the following hypothesis test at the $\alpha = .01$ level of significance.

$$H_0 : \mu_d = 0$$
$$H_1 : \mu_d \neq 0$$

Sample 1	**28**	**29**	**22**	**25**	**26**	**29**	**27**	**24**	**27**	**28**
Sample 2	**34**	**30**	**31**	**26**	**31**	**30**	**31**	**32**	**29**	**37**
Difference	**-6**	**-1**	**-9**	**-1**	**-5**	**-1**	**-4**	**-12**	**-2**	**-9**

$$\bar{d} = \frac{-50}{10} = -5$$

$$s_d = \sqrt{\frac{140}{9}} = 3.94$$

$$t = \frac{-5}{\frac{3.94}{\sqrt{10}}} = -4.01304...$$

Is $-4.01304... \leq -3.25$**?**