

Present Value of an Annuity:

The present value of an annuity is the amount of money that you could deposit right now into the account that would produce the same amount of money as the annuity does after n payments.

$$\underbrace{PV(1+i)^n}_{\text{money produced from the present value}} = \underbrace{Pmt \left[\frac{(1+i)^n - 1}{i} \right]}_{\text{money produced from the } n \text{ annuity payments}}$$

So

$$PV = Pmt \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right]$$

Or

$$PV = Pmt \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

Example:

Find the present value of an annuity with monthly payments of \$500 into an account paying 3.1% compounded monthly for 6 years.

$$PV = 500 \left[\frac{1 - \left(1 + \frac{.031}{12}\right)^{-72}}{\frac{.031}{12}} \right] = \boxed{\$32,811.73}$$

Another interpretation of the present value is that it's the amount of money you can deposit now into the account that will allow you to make n equal periodic withdrawals from the account so that the n^{th} withdrawal empties the account.

Example:

How much money should you deposit now into an account paying 2.7% compounded semi-annually that will allow you to make semi-annual withdrawals of \$2,000 for the next 10 years with your last withdrawal emptying the account?

$$PV = 2000 \left[\frac{1 - \left(1 + \frac{.027}{2}\right)^{-20}}{\frac{.027}{2}} \right] = \$34,850.43$$

Loan Repayment(Amortization):

When you borrow money, you are acting like a bank that is holding a deposit that is earning compound interest. The company that loaned you the money will make n equal periodic withdrawals(your payments) until the money they deposited with you is zeroed out by the last withdrawal(your last payment).

We'll just solve the present value formula for Pmt to find a formula for your payment amount in repaying the loan.

$$PV = Pmt \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

$$Pmt = PV \left[\frac{i}{1 - (1 + i)^{-n}} \right]$$

Examples:

- 1. You borrow \$20,000 at 3.9% compounded monthly for 5 years to buy a car. What's your monthly payment? How much extra in interest will you pay on this loan?**

$$Pmt = 20,000 \left[\frac{\frac{.039}{12}}{1 - \left(1 + \frac{.039}{12}\right)^{-60}} \right] = \boxed{\$367.43}$$

Your interest portion is $\boxed{\$2,045.80}$.



- 2. You borrow \$150,000 at 3.1% compounded monthly for 30 years to buy a house. What's your monthly payment? How much extra in interest will you pay on this loan?**

$$Pmt = 150,000 \left[\frac{\frac{.031}{12}}{1 - \left(1 + \frac{.031}{12}\right)^{-360}} \right] = \boxed{\$640.52}$$

Your interest portion is $\boxed{\$80,587.20}$.



The interest portion is the 360 payments of \$640.52 minus the loan amount of \$150,000.

Scientific Calculator Advice:

1. How much money should you deposit now into an account paying 2.7% compounded semi-annually that will allow you to make semi-annual withdrawals of \$2,000 for the next 10 years with your last withdrawal emptying the account?

The formula that we will use to find the amount of money is $PV = Pmt \left[\frac{1 - \left(1 + \frac{r}{m}\right)^{-n}}{\frac{r}{m}} \right]$. We need to plug in the

correct values and get the calculator to cooperate. Let's start with plugging in the correct values: The payment amount is \$2,000. The annual interest rate as a decimal is .027. The number of payments/compounding-periods in one year is 2. The total number of payments/compounding-periods is 20. Therefore,

$PV = 2,000 \left[\frac{1 - \left(1 + \frac{.027}{2}\right)^{-20}}{\frac{.027}{2}} \right]$. Now for the calculator. Using a standard scientific calculator, I'd start by typing in

.027, pressing $\div$, typing 2 and pressing =. Then I would press the + key, type 1 and press =. We have to raise this to the negative 20th power, so I'd press either ^ or the x^y key (depending on the calculator), type in 20, press the change sign key and press =. Next, I'd press the change sign key, press the + key, type 1 and press =. Then I'd press $\div$, type .027 and press =. Then I'd press $\times$, type 2 and press =. Finally, I'd press $\times$, type 2000 and press =. You should be looking at 34850.4191 on your calculator display. This needs to be converted into dollar and cent format rounded to the nearest penny, which would be \$34,850.42.

2. You borrow \$20,000 at 3.9% compounded monthly for 5 years to buy a car. What's your monthly payment?

The formula that we will use to find the payment amount is $Pmt = PV \left[\frac{\frac{r}{m}}{1 - \left(1 + \frac{r}{m}\right)^{-n}} \right]$. We need to plug in the

correct values and get the calculator to cooperate. Let's start with plugging in the correct values: The present value amount is \$20,000. The annual interest rate as a decimal is .039. The number of payments/compounding-periods in one year is 12. The total number of payments/compounding-periods is 60. Therefore,

$Pmt = 20,000 \left[\frac{\frac{.039}{12}}{1 - \left(1 + \frac{.039}{12}\right)^{-60}} \right]$. Now for the calculator. Using a standard scientific calculator, I'd start by typing

in .039, pressing $\div$, typing 12 and pressing =. Then I would press +, type 1 and press =. We have to raise this to the negative 60th power, so I'd press either ^ or the x^y key (depending on the calculator), type in 60, press the change sign key and press =. Next, I'd press the change sign key, press the + key, type 1 and press =. So far we've calculated the value of the denominator, but the calculator doesn't know that this value is the denominator.

To move this value into the denominator, we need to press the either the $\frac{1}{x}$ or the x^{-1} key (depending on the calculator). Then I'd press $\times$, type .039 and press =. Then I'd press $\div$, type 12 and press =. Finally, I'd press $\times$, type 20,000 and press =. You should be looking at 367.4285788 on your calculator display. This needs to be converted into dollar and cent format rounded to the nearest penny, which would be \$367.43.