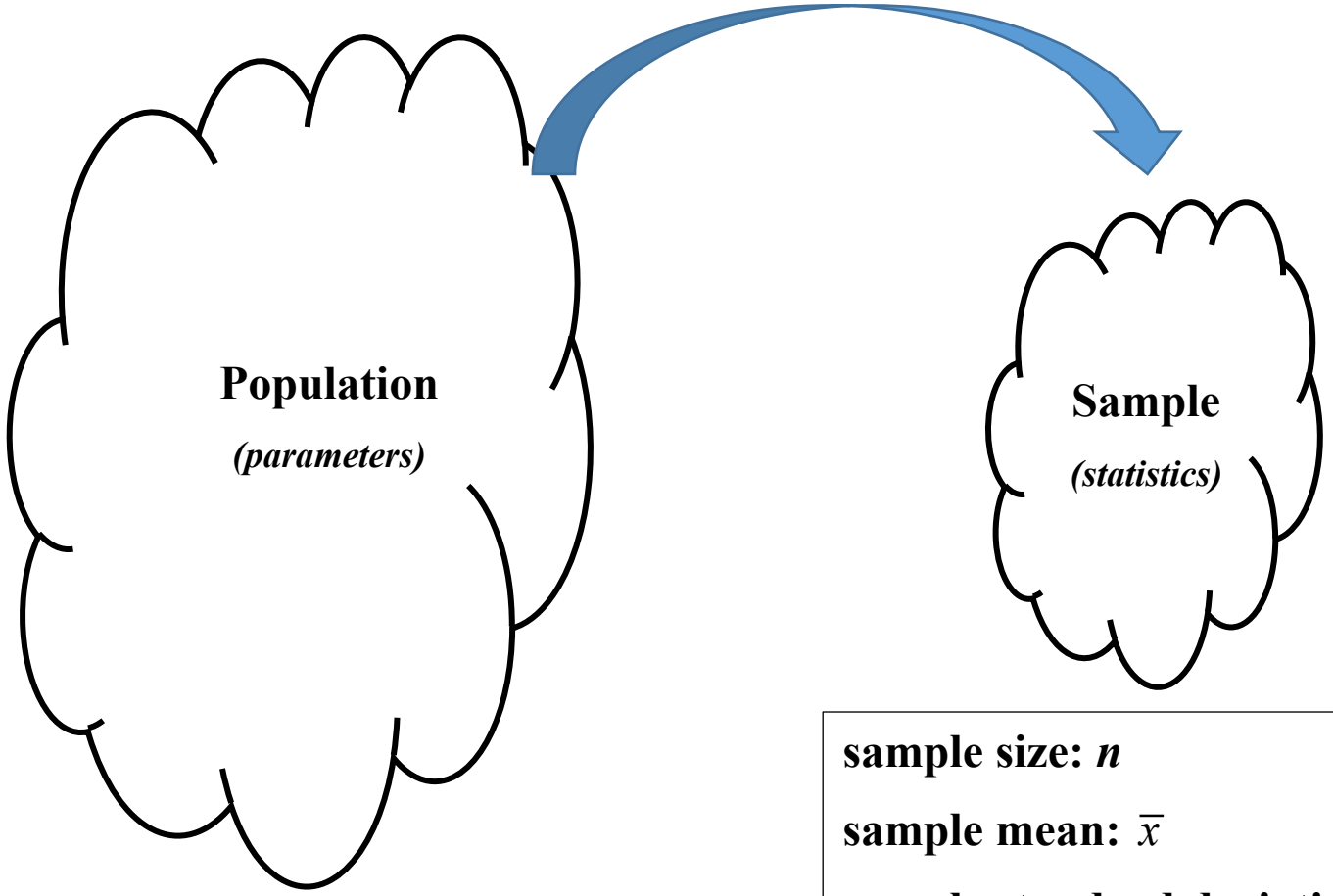
Populations and Samples:



population size: N

population mean: μ

population standard deviation: σ

sample size: n

sample mean: $\bar{x}$

sample standard deviation: s

The goal in inferential statistics is to use information from the sample*(statistic)* to make a prediction about the population*(parameter)*. In order to do this, we have to know about the distribution of the sample, called the sampling distribution.

Known Properties of the Sampling Distribution of the Sample Mean:

If the sampling is done *with* replacement, then

1. The mean of the sample means, $\mu_{\bar{x}}$, is the same as the population mean, μ, i.e.

$\mu_{\bar{x}} = \mu$.

2. The standard deviation of the sample means, $\sigma_{\bar{x}}$, is the population standard deviation, σ, divided by the square-root of the sample size, i.e. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.

If the sampling is done *<u>without</u>* replacement, then

1. The mean of the sample means, $\mu_{\bar{x}}$, is the same as the population mean, μ, i.e.

$$\mu_{\bar{x}} = \mu.$$

2. The standard deviation of the sample means, $\sigma_{\bar{x}}$, has a different formula:

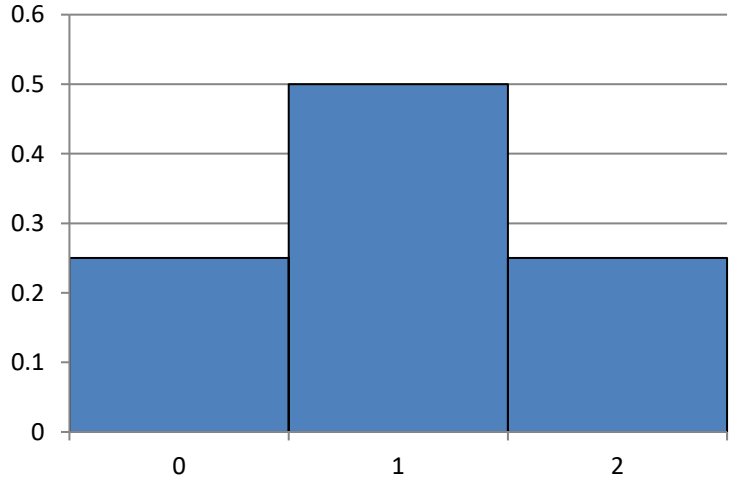
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}.$$

Example:

Here is the frequency distribution for a population.

X	frequency	relative frequency
0	**6**	$\frac{1}{4}$
1	**12**	$\frac{1}{2}$
2	**6**	$\frac{1}{4}$

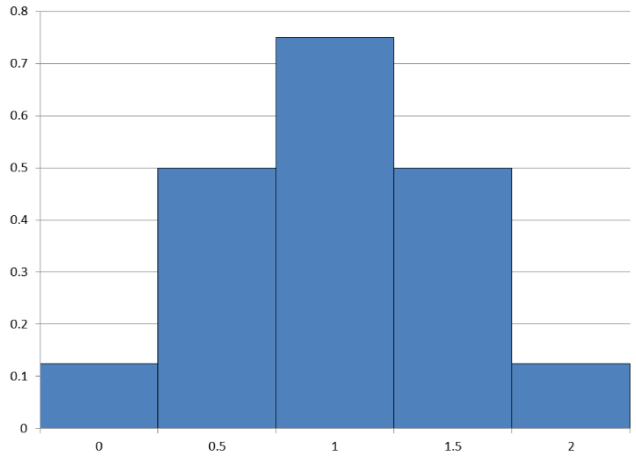
Here is the relative frequency histogram for the population.



The population mean is 1, and the population standard deviation is $\frac{1}{\sqrt{2}}$.

If we take samples of size 2(with replacement), then the probability distribution of the sample means is

$\bar{x}$	**Samples**	$P(\bar{x})$
0	**0,0**	$\frac{1}{4}\cdot\frac{1}{4}=\boxed{\frac{1}{16}}$
$\frac{1}{2}$	**0,1** **1,0**	$2\cdot\frac{1}{4}\cdot\frac{1}{2}=\boxed{\frac{1}{4}}$
1	**1,1** **0,2** **2,0**	$\frac{1}{2}\cdot\frac{1}{2}+2\cdot\frac{1}{4}\cdot\frac{1}{4}=\boxed{\frac{3}{8}}$
$\frac{3}{2}$	**1,2** **2,1**	$2\cdot\frac{1}{2}\cdot\frac{1}{4}=\boxed{\frac{1}{4}}$
2	**2,2**	$\frac{1}{4}\cdot\frac{1}{4}=\boxed{\frac{1}{16}}$



Let's find the mean of the sample means, and compare it to the population mean.

$\bar{x}$	$P(\bar{x})$	$\bar{x} \cdot P(\bar{x})$
0	$\frac{1}{16}$	**0**
$1/2$	$\frac{1}{4}$	$\frac{1}{8}$
1	$\frac{3}{8}$	$\frac{3}{8}$
$3/2$	$\frac{1}{4}$	$\frac{3}{8}$
2	$\frac{1}{16}$	$\frac{1}{8}$
Total		$\frac{8}{8}=1$

$\mu_{\bar{x}} =$ **1, which is the same as the population mean.**

Let's find the standard deviation of the sample means, and compare it to the population standard deviation.

$\bar{x}$	$(\bar{x}-\mu_{\bar{x}})^2$	$P(\bar{x})$	$(\bar{x}-\mu_{\bar{x}})^2 \cdot P(\bar{x})$
0	1	$\frac{1}{16}$	$\frac{1}{16}$
$1/2$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{16}$
1	**0**	$\frac{3}{8}$	**0**
$3/2$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{16}$
2	**1**	$\frac{1}{16}$	$\frac{1}{16}$
Total			$\frac{4}{16}=\frac{1}{4}$

$\sigma_{\bar{x}}=\sqrt{\frac{1}{4}}=\frac{1}{2}$**, which is the population standard deviation divided by** $\sqrt{2}$**.**

Normal Populations:

If the population is normal with mean of μ and standard deviation of σ, then any kind of sampling will have the sample means normally distributed with $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.

Example:

A sample of size 16 is to be drawn from a normal population with a mean of 20 and a standard deviation of 5. Calculate the following probabilities of the sample mean.

1. $P(\bar{x} < 18)$

$$P\left(Z < \frac{18-20}{\frac{5}{4}}\right)$$

$$P(Z < -1.6) = .0548$$

2. $P(\bar{x} > 21)$

$$P\left(Z > \frac{21-20}{\frac{5}{4}}\right)$$

$$P(Z > .8) = .2119$$

3. $P(18 < \bar{x} < 21)$

$$P\left(\frac{18-20}{\frac{5}{4}} < Z < \frac{21-20}{\frac{5}{4}}\right)$$

$$P(-1.6 < Z < .8) = .7881 - .0548 = .7333$$

Central Limit Theorem:

If the sample size is larger than 30, $n > 30$, from a population with mean of μ and standard deviation of σ, then the sample means will be approximately normally distributed with $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.

Example:

A sample of size 36 is to be drawn from a population with a mean of 40 and a standard deviation of 12. Calculate the following probabilities of the sample mean.

1. $P(\bar{x} < 37)$

$$P\left(Z < \frac{37-40}{\frac{12}{6}}\right)$$

$$P(Z < -1.5) = .0668$$

2. $P(\bar{x} > 42)$

$$P\left(Z > \frac{42-40}{\frac{12}{6}}\right)$$

$$P(Z > 1) = .1587$$

3. $P(38 < \bar{x} < 41)$

$$P\left(\frac{38-40}{\frac{12}{6}} < Z < \frac{41-40}{\frac{12}{46}}\right)$$

$$P(-1 < Z < .5) = .6915 - .1587 = .5328$$