Name:_____

Student ID:_____

Section:_____

Instructor:_____

Math 2318 (Linear Algebra)
Practice Exam 2

August 8, 2016

Instructions:

- Work on scratch paper will not be graded.
- For questions 21 to 27, show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer.
- Please write neatly. If I cannot read your handwriting, you will not receive credit.
- Simplify your answers as much as possible. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

For Instructor use only.

#	Possible	Earned
MC	60	
21	5	
22	5	
23	5	
Sub	75	

#	Possible	Earned
24	5	
25	10	
26	5	
27	5	
Sub	25	
Total	100	

Multiple Choice. Circle the correct answer for each question. Circle one choice only.

1. Find the adjoint of the matrix

$$A = \begin{pmatrix} -3 & -5 & -7 \\ 2 & 4 & 3 \\ 0 & 1 & -1 \end{pmatrix}$$

a) b) c)

d) e) f)

2. Use Cramer's Rule to solve the system of linear equations.

$$\begin{aligned} 4x_1 - x_2 + x_3 &= -5 \\ 2x_1 + 2x_2 + 3x_3 &= 10 \\ 5x_1 - 2x_2 + 6x_3 &= 1 \end{aligned}$$

a) b) c)

d) e) f)

3. Find the volume of the tetrahedron with the given vertices $(3, -1, 1)$, $(4, -4, 4)$, $(1, 1, 1)$ and $(0, 0, 1)$.

a) b) c)

d) e) f)

4. Which of the following sets is a vector space? Choose the best answer.

   a) The set of all fifth-degree polynomial with the standard operations.

   b) The set of all 2-by-2 nonsingular matrices with the standard operations.

   c) The set $\{(x, y)$ such that $x \geq 0$ and y is a real number$\}$.

   d) The set of all polynomials of degree four or less with the standard operations.

   e) A and D.

   f) B and C.

5. Which of the following sets is NOT a subspace of the vector space?

   a) W is set of all 2-by-2 matrices of the form

   $$\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$$

   and $V = M_{2,2}$.

   b) W is the set of all odd continuous functions $f(-x) = -f(x)$ and $V = C(-\infty, \infty)$ the set of all continuous functions on $(-\infty, \infty)$.

   c) W is the set of all vectors in $\mathbb{R}^3$ whose third component is -1 and $V = \mathbb{R}^3$.

   d) W is the set of all n-by-n invertible matrices and $V = M_{n,n}$.

   e) A and B.

   f) C and D.

6. Which of the following sets is linearly independent?

   a) $S = \{(3, -6), (-1, 2)\}$

   b) $S = \{(1, 0), (1, 1), (2, -1)\}$

   c) $S = \{(1, 0, 0), (0, 4, 0), (0, 0, -6), (1, 5, -3)\}$

   d) $S = \{(1, 2, 3), (3, 2, 1), (0, 0, 1)\}$

   e) $S = \{2 - x, 2x - x^2, 6 - 5x + x^2\}$

   f) $S = \{(1, 2, 3, 4), (1, 0, 1, 2), (1, 4, 5, 6)\}$

7. Which of the following sets is a basis for $\mathbb{R}^2$?

   a) $\{(1, 2), (1, 0), (0, 1)\}$

   b) $\{(-4, 5), (0, 0)\}$

   c) $\{(-1, 2)\}$

   d) $\{(3, -2), (4, 5)\}$

   e) $\{(4, -3), (8, -6)\}$

   f) $\{1, x, x^2\}$

8. Find the dimension of the vector space $M_{4,6}$

   a) b) c)

   d) e) f)

9. Find the dimension of the vector space of all 4-by-4 diagonal matrices?

   a) b) c)

   d) e) f)

10. Find the rank of the matrix
$$A = \begin{pmatrix} -2 & -4 & 4 & 5 \\ 3 & 6 & -6 & -4 \\ -2 & -4 & 4 & 9 \end{pmatrix}.$$

a) b) c)

d) e) f)

11. Find the nullity (the dimension of the nullspace) of the matrix
$$A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & -1 & 4 \\ 4 & 3 & -2 \end{pmatrix}.$$

a) b) c)

d) e) f)

12. Find the coordinate matrix of $x = (3, 19, 2)$ relative to the basis $B' = \{(8, 11, 0), (7, 0, 10), (1, 4, 6)\}$.

a) b) c)

d) e) f)

13. Find the transition matrix from B to B' where $B = \{(1, 3), (-2, -2)\}$ and $B' = \{(-12, 0), (4, 4)\}$.

a) b) c)

d) e) f)

14. Let A be the coordinate matrix of $p = 2x^3 + x^2 + 11x + 4$ and let B be the coordinate matrix of $q = x^3 - 2x^2 + 5x + 1$ relative to the standard basis $\{1, x, x^2, x^3\}$ of P_3. Find the product $B^T A$.

a) b) c)

d) e) f)

15. Find a basis for the column space of the matrix
$$A = \begin{pmatrix} 1 & 2 & 4 \\ -1 & 2 & 1 \end{pmatrix}.$$

a) b) c)

d) e) f)

16. Let A be an 7-by-6 matrix whose rank is 5. Find the dimension of the solution space of $Ax = 0$.

a) b) c)

d) e) f)

17. True or False. If U and W are both subspaces of a vector space V, then the intersection of U and W is also a subspace of V.

a) True. b) False.

18. True or False. If the dimension of a vector space V is n, then any set of $n + 1$ vectors in V must be linearly dependent.

a) True. b) False.

19. True or False. If the rank of an 5-by5 matrix A is 5, then A is singular.

a) True. b) False

20. True or False. The column space of a matrix A is equal to the row space of A^T.

a) True. b) False.

Free response: Show all work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer. Please write neatly. Scratch work will not be graded.

21. (5 points) Solve the system of linear equations for x and y in terms of k using Cramer's Rule. (Any other method will receive no credits). For which value(s) of k will the system be consistent?

$$kx + (1-k)y = 1$$
$$(1-k)x + ky = 3.$$

22. (5 points) Rather than use the standard definitions of addition and scalar multiplication in $\mathbb{R}^3$, suppose these two operations are defined as fillows.

$$(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2 + 1, y_1 + y_2 + 1, z_1 + z_2 + 1)$$
$$c(x_1, y_1, z_1) = (cx + c - 1, cy + c - 1, cz + c - 1)$$

With these new definitions, is $\mathbb{R}^3$ a vector space? If your answer is yes, prove it by proving that all the axioms of a vector space are satisfied. If your answer is no, prove it by pointing out one axiom that is not satisfied and provide an example to demonstrate that the axiom is not satisfied.

23. (5 points) Let A be a fixed m-by-n matrix. Prove that the set

$$W = \{x \text{ in } \mathbb{R}^n \text{ such that } Ax = 0.\}$$

Prove that W is a subspace of $\mathbb{R}^n$

24. (5 points) Consider the subspace $W = \{(2s-t, s, t, s) : s \text{ and } t \text{ are real numbers}\}$ of $\mathbb{R}^4$. Find a basis for W and find the dimension of W.

25. (10 points) Let $B = \{(1,0,2),(0,1,3),(1,1,1)\}$ and $B' = \{(2,1,1),(1,0,0),(0,2,1)\}$.

(a) Find the transition matrix P^{-1} from B to B'.

(b) Suppose that the coordinate matrix of x relative to the basis B is

$$[x]_B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

Find $[x]_{B'}$ the coordinate matrix of x relative to the basis B'

26. (5 points)For which values of t is the set $S = \{(t,1,1),(1,0,1),(1,1,3t)\}$ is linearly independent.

27. (5 points) Let A be a nonsingular matrix of order 3. Prove that if $\{v_1, v_2, v_3\}$ is a linearly independent set in $\mathbb{R}^3$, then the set $\{Av_1, Av_2, Av_3\}$ is also linearly independent.

END OF EXAM