MATH 2318 LEARNING GOALS

1. Larson's Chapter 1 - Sections 1.1 and 1.2

1.1. Introduction to Systems of Linear Equations.

(1) Determine whether a given equation is linear. **Ex**: 1, 4, 5, 6.
(2) Find a parametric representation of the solution set of a linear equation. **Ex**: 10.
(3) Graph and solve a system of linear equations in 2 variables. **Ex**: 11, 13.
(4) Solve a system of linear equations by using back substitution or Gaussian elimination. **Ex**: 28, 47, 51, 71, 77, 79.

1.2. Gaussian Elimination and Gauss-Jordan Elimination.

(1) Know the elementary row operations, the row-echelon form and reduced row-echelon form of a matrix. **Ex**: 8, 9, 17, 19, 20, 21, 22, 53, 54, 57, 58.
(2) Solve a system of linear equations by applying Gaussian elimination with back-substitution or Gauss-Jordan elimination to the augmented matrix. **Ex**: 31, 37, 47.

2. Chapter 2 - Sections 2.1, 2.2, 2.3 and 2.4

2.1. Operations with Matrices.

(1) Be able to perform basic operations with matrices. **Ex**: 4, 7, 13, 15, 17, 20, 23, 24, 34, 36, 38, 57, 59, 63, 67, 75, 77.
(2) Solve matrix equations of the form $A\mathbf{x} = \mathbf{0}$. **Ex**: 39.
(3) Solve matrix equations of the form $A\mathbf{x} = \mathbf{b}$ and use the result to **Ex**press $\mathbf{b}$ as a linear combination of the columns of A. **Ex**: 51.

2.2. Properties of Matrix Operations.

(1) Understand and apply the properties of matrix addition, scalar multiplication and zero matrices to solve problems. **Ex**: 11, 13c, 17, 49.
(2) Understand and apply the properties of matrix multiplication and the identity matrix to solve problems. **Ex**: 23, 26, 27, 30, 31, 51, 53.
(3) Define the transpose of a matrix, symmetric matrices and solve problems involving these concepts. **Ex**: 40, 44, 65, 67, 68, 74.

2.3. The Inverse of a Matrix.

(1) Verify two matrices are inverses of one another. **Ex**: 3.
(2) Understand and be able to find the inverse of a matrix. **Ex**: 9, 17, 41, 55, 57, 67, 69, 71, 72.
(3) Solve system of linear equations using an inverse matrix. **Ex**: 47.

2.4. Elementary Matrices.

(1) Understand properties of elementary matrices, especially the relationship between elementary matrices and elementary row operations. **Ex**: 9, 10, 11, 12, 13, 27, 31, 57, 58.
(2) Understand the Theorem on the equivalent conditions for a matrix to be invertible. **Ex**: 59.
(3) Find the LU-factorization of a matrix and solve a linear system using LU-factorization. **Ex**: 43, 45.

3. Chapter 3 - Sections 3.1, 3.2, 3.3 and 3.4

3.1. **The determinant of a matrix.**

(1) Find the determinant of a 2-by-2 matrix. **Ex**: 6, 45, 49, 62, 63, 64.
(2) Find the determinant by cofactor expansion. **Ex**: 21, 31, 32.
(3) Use the shortcut to find the determinant of a 3-by-3 matrix. **Ex**: 33.
(4) Find the determinant of a triangular matrix. **Ex**: 42.

3.2. **Evaluation of a determinant using elementary operations.**

(1) Understand and apply the effects of elementary row operations on determinants and conditions that yield a zero determinant. **Ex**: 3, 4, 5, 7, 9, 12, 15, 19.
(2) Use elementary row or column operations to find determinants. **Ex**: 27, 33.

3.3. **Properties of Determinants.**

(1) Understand the properties of determinants and apply them to solve problems. **Ex**: 3, 8, 13, 25, 27, 37, 57, 66, 69, 80.
(2) Understand the equivalent conditions of a nonsingular matrix and apply them to solve problems. **Ex**: 33, 58, 68.

3.4. **Applications of Determinants.**

(1) Find the adjoint of a matrix and use the adjoint to find the inverse. **Ex**: 1, 5.
(2) Use Cramer's rule to solve a system of linear equations. **Ex**: 17, 29, 35.
(3) Use determinants to find the area of a triangle, the volume of a tetrahedron, and to determine whether 3 given points are collinear or 4 given points are coplanar. **Ex**: 39, 43, 49, 55.

4. Chapter 4 - Sections 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7

4.1. **Vectors in $\mathbb{R}^n$.**

(1) Understand the basic operations of vectors in $\mathbb{R}^n$. **Ex**: 7, 17, 22, 23, 27, 41, 45.
(2) Understand the properties of vector addition and scalar multiplication in $\mathbb{R}^n$. **Ex**: 61, 62, 63.

4.2. **Vectos Spaces.**

(1) Understand the definition of a vector space and be able to determine whether a set with a given operation is a vector space. **Ex**: 13, 19, 25, 27, 29, 35a, 35b, 44.

4.3. **Subspaces of Vector Spaces.**

(1) Understand the definition of a subspace of a vector space and be able to determine whether a subset is a subspace. **Ex**: 1, 4, 7, 13, 15, 19, 27, 29, 33, 34, 37, 41, 43, 44, 54.

4.4. **Spanning sets and linear independence.**

(1) Write a vector as a linear combination of given vectors. **Ex**: 3a, 3b.
(2) Understand the definition of spanning set and the span of a set. Determine whether a given set spans a vector space. **Ex**: 9, 17, 25.
(3) Determine whether a set of vectors is linearly independent. **Ex**: 43, 45, 59, 60, 61.

4.5. **Basis and dimension.**

(1) Understand the definition of the basis of a vector space. Determine whether a given set is a basis for a vector space. **Ex**: 2, 3, 6, 7, 18, 19, 21, 30, 32, 35, 48.
(2) Understand the definition of the dimension of a vector space. Find the dimension of a vector space. **Ex**: 52, 54, 55, 57, 63, 66, 71, 72.

4.6. **Rank of a matrix and system of linear equations.**

(1) Find the rank, a basis for the row space, a basis for the column space of a given matrix. **Ex**: 9, 13, 21.
(2) Find the basis for and the dimension of the solution space of $A\mathbf{x} = \mathbf{0}$ or the nullspace of a matrix A. **Ex**: 31, 33, 41.
(3) Understand the relationship between the rank and the nullity of a matrix and the equivalent conditions for a square matrices. **Ex**: 69, 70, 71.

4.7. **Coordinates and change of basis.**

(1) Find the coordinate matrix of a vector relative to a basis. **Ex**: 5, 7, 11, 13, 43.
(2) Find the transition matrix from a basis to another. **Ex**: 17, 21, 37, 51, 52.

5. Chapter 5 - Sections 5.1, 5.2, 5.3, 5.4

5.1. **Length and dot produtct in $\mathbb{R}^n$.**

(1) Find the length (norm) of a vector and the distance between two vectors. **Ex**: 7, 12, 15, 18, 21.
(2) Find the dot product of two vectors and angle between two vectors. Determine orthogonality. **Ex**: 25, 27, 43, 45, 47, 55, 81, 83.
(3) Understand the Cauchy-Schwarz inequality, the triangle inequality and the Pythagorean theorem and solve related problems. **Ex**: 35, 59, 64.

5.2. **Inner product spaces.**

(1) Understand the definition of an inner product space. Verify that a function is an inner product. Find the norm (length) of a vector, distance and angle between two vectors; determine orthogonality in an inner product space. **Ex**: 5, 14, 24, 33, 35, 39, 47, 49, 51, 67, 93, 95.
(2) Find the orthogonal projection of an vector onto another vector in an inner product space. **Ex**: 69, 73, 77, 90.

5.3. **Orthonormal bases: Gram-Schmidt Process.**

(1) Determine whether a given set of vectors in an inner product space is orthogonal or orthonormal. **Ex**: 5, 9, 11, 17, 48, 58.
(2) Find the coordinates of a vector relative to a given orthonormal basis. **Ex**: 23, 25.
(3) Use the Gram-Schmidt orthonomalization process to find orthonormal bases. **Ex**: 27, 35, 57, 59, 61.

5.4. **Mathematical models and least squares analysis.**

(1) Determine whether two sets of vectors are orthogonal. **Ex**: 5.
(2) Find the orthogonal complement of a subspace in an inner product space. **Ex**: 9, 27, 41.
(3) Find the projection of a vector onto a subspace. **Ex**: 17.
(4) Find the bases for the four subspace of a given matrix. **Ex**: 19.
(5) Solve the least square problems. **Ex**: 23, 25, 31, 42.

6. Chapter 6 - Sections 6.1, 6.2, 6.3, 6.4

6.1. **Introduction to linear transformations.**

(1) Find the image or preimage of a vector under a transformation. **Ex**: 2, 23, 29, 45, 55.
(2) Determine whether a given function is a linear transformation. **Ex**: 11, 13, 17, 19, 21, 71.
(3) Understand and apply the properties of linear transformation to solve problems. **Ex**: 75, 83.

6.2. **The kernel and range of a linear transformation.**

(1) Find the kernel, the range, a basis for the kernel/range, the nullity and the rank of a given linear transformation. **Ex**: 2, 5, 9, 13, 21, 55.
(2) Understand the rank-nullity theorem, onto and one-to-one linear transformations and solve related problems. **Ex**: 31, 42, 45, 49, 57

6.3. **Matrices for linear transformations.**

(1) Find the standard matrix for a given linear transformation. Use the standard matrix to find the image of a given vector. **Ex**: 3, 9, 11, 13, 19, 21.
(2) Find the standard matrix for a composition of transformations. **Ex**: 29, 54.
(3) Determine whether a linear transformation is invertible and find its inverse if possible. **Ex**: 35.
(4) Find the transformation matrix relative to nonstandard bases. **Ex**: 37, 43, 53.

6.4. **Transition matrices and similarity.**

(1) Find the matrix for a linear transformation $T : V \longrightarrow V$ relative to any given basis and use that matrix to solve problems. **Ex**: 7, 9, 13, 19, 37.
(2) Understand similar matrices and solve related problems. **Ex**: 15, 21, 27.

7. Chapter 7 - Sections 7.1, 7.2, 7.3

7.1. **Eigenvalues and eigenvectors.**

(1) Verify or determine the eigenvalues and corresponding eigenvectors of a matrix. Solve related problems. **Ex**: 5, 17, 23, 41, 60, 65.
(2) Find a basis and the dimension of the eigenspace corresponding to an eigenvalue. **Ex**: 43, 45, 69.

7.2. **Diagonalization.**

(1) Determine whether a matrix is diagonalizable. **Ex**: 3, 15, 19, 23, 26.
(2) Diagonalize a diagonalizable matrix. **Ex**: 7, 11, 37, 38, 43, 45.

7.3. **Symmetric matrices and orthogonal diagonalization.**

(1) Find the eigenvalues and corresponding eigenspaces of a symmetric matrix. **Ex**: 11, 15, 34.
(2) Determine whether a matrix is orthogonal. **Ex**: 23, 25, 54.
(3) Orthogonal diagonalize a symmetric matrix. **Ex**: 41, 47, 53.