2.5 - Related Rates

Idea: 2 quantities. Each quantity is a function of time.

x, y : x, y are related.

x, y : functions of time t.

We know how one quantity is changing with respect to time; that is, we know the rate of change of one quantity with respect to time. We'd like to calculate how the other quantity is changing.

Jul 26-12:01 PM

$\boxed{x^2 + y^2 = 25}$ ← relationship between x and y.

Both x and y are functions of time.

$\boxed{\frac{dx}{dt} = 10}$. Find $\frac{dy}{dt}$ when $\boxed{x = 3}$

$x^2+y^2=25$. Take derivative with respect to time of both sides

$$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(25)$$

Jul 26-12:06 PM

$$\frac{d}{dt}(x^2+y^2) = 0$$

$u = x^2$

$u — x — t$

$\frac{du}{dx} \cdot \frac{dx}{dt}$

$$\frac{d}{dt}(x^2) + \frac{d}{dt}(y^2) = 0$$

$$\boxed{2\overset{3}{x}\overset{10}{\frac{dx}{dt}} + 2\overset{4}{y}\overset{?}{\frac{dy}{dt}} = 0}$$

Find $\frac{dy}{dt}$? Given $\frac{dx}{dt} = 10$; $x = 3$

$x^2+y^2=25$. When $x=3$. $9+y^2=25$

$y^2=16$; $y = \pm 4$

$\boxed{y=4.}$

$\underbrace{2\cdot 3\cdot 10}_{60} + 2\cdot 4 \cdot \frac{dy}{dt} = 0$

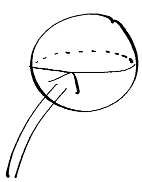
$60 + 8\frac{dy}{dt} = 0$

Jul 26-12:10 PM

$$8\frac{dy}{dt} = -60$$

$$\frac{dy}{dt} = \frac{-60}{8} = \frac{-15}{2}$$

Jul 26-12:16 PM



air

balloon

r: radius of balloon

$\frac{dr}{dt} = \boxed{3}$ inches per minute.

Find how the volume of the balloon is changing as $\boxed{r = 12}$ inches

V: volume. $\frac{dV}{dt} = ?$

$$V = \frac{4}{3}\pi r^3$$

Jul 26-12:17 PM

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2 \cdot \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} \;;$$

$\frac{dr}{dt} = 3$; $r = 12$

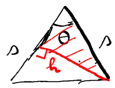
$$\frac{dV}{dt} = 4\pi\cdot(12)^2\cdot 3 = \boxed{4\pi\cdot 144\cdot 3}$$

inch^3 per minute.

Jul 26-12:20 PM

2.6.12

① Show that the area of this triangle is



Area $= \frac{1}{2} s^2 \cdot \sin\theta$

Area $= \frac{1}{2}$ base $\cdot$ height $= \frac{1}{2} \cdot s \cdot h$

$\frac{h}{s} = \sin\theta \implies h = \sin\theta \cdot s$

$\boxed{\text{Area} = \frac{1}{2} \cdot s \cdot s \cdot \sin\theta = \frac{1}{2} s^2 \sin\theta}$

② $\frac{d\theta}{dt} = \frac{1}{2}$ radian per minute. Find $\frac{d\,\text{Area}}{dt}$ when $\theta = \pi/6$

Jul 26-12:24 PM

$A = \frac{1}{2} s^2 \cdot \sin\theta$. Given $\boxed{\frac{d\theta}{dt} = \frac{1}{2}}$.

Find $\boxed{\frac{dA}{dt}}$ when $\theta = \frac{\pi}{6}$.

$\frac{dA}{dt} = \frac{d}{dt}\left(\left(\frac{1}{2} s^2\right) \sin\theta\right)$

$\frac{dA}{dt} = \frac{1}{2} s^2 \frac{d}{dt}(\sin\theta)$

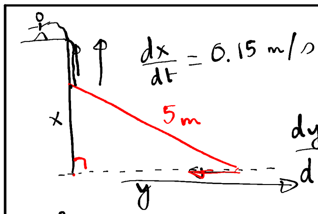
$\frac{dA}{dt} = \frac{1}{2} s^2 \cos\theta \cdot \frac{d\theta}{dt}$

$\frac{d\theta}{dt} = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

$\frac{dA}{dt} = \frac{1}{2} s^2 \cdot \cos\frac{\pi}{6} \cdot \frac{1}{2} = \frac{1}{2} s^2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{\sqrt{3}}{8} \cdot s^2$

Jul 26-12:32 PM



$\frac{dx}{dt} = 0.15$ m/s

$\frac{dy}{dt} = ?$ when $y = 2.5$ m from the building

$x^2 + y^2 = 25$

$2\boxed{x}\boxed{\frac{dx}{dt}} + 2\boxed{y}\frac{dy}{dt} = 0$

x: 4.33, $\frac{dx}{dt}$: 0.15, y: 2.5

when $y = 2.5$

$x^2 + (2.5)^2 = 25$

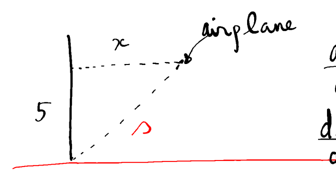
$x^2 = 25 - (2.5)^2$

$x^2 = 18.75$;

$x \approx 4.33$

$2 \cdot (4.33) \cdot (0.15) + 2 \cdot (2.5) \frac{dy}{dt} = 0$

Jul 26-12:37 PM



$\frac{ds}{dt} = 240$ miles per hour.

$\frac{dx}{dt} = ?$ when $s = 10$ miles.

$x^2 + 25 = s^2$

$2\boxed{x}\boxed{\frac{dx}{dt}} = 2\boxed{s}\boxed{\frac{ds}{dt}}$

x: $\sqrt{75}$, $\frac{dx}{dt}$: ?, s: 10, $\frac{ds}{dt}$: 240

$x^2 + 25 = 100$

$x^2 = 75$;

$x = \sqrt{75}$

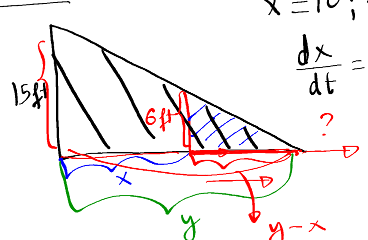
$2 \cdot \sqrt{75} \cdot \frac{dx}{dt} = 2 \cdot 10 \cdot 240$; $\boxed{\frac{dx}{dt} = \frac{4800}{2\sqrt{75}}}$

Jul 26-12:44 PM

2.6.29

$x = 10$; what is $\frac{dy}{dt} = ?$

$\frac{dx}{dt} = 5$ ft/s



$\boxed{\frac{15}{y} = \frac{6}{y-x}} \implies 15(y-x) = 6y$

$15y - 15x = 6y$

$\boxed{9y = 15x}$

Jul 26-12:51 PM

$9y = 15x$

$3y = 5x$

$3\frac{dy}{dt} = 5\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{5}{3}\frac{dx}{dt} = \boxed{\frac{25}{3}}$ ft/s

$\frac{dx}{dt} = 5$ ft/s

$x = 10$

Jul 26-12:58 PM

Step1: Identify the quantities that are changing with respect to time.

Step 2: Find a relation between these quantities

Step3: Differentiate w.r.t. time. Plug in the given data.

Jul 26-1:00 PM