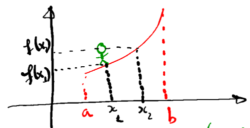
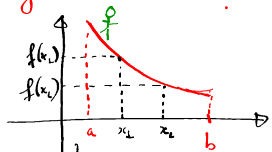
3.3. Increasing / Decreasing functions and the first derivative test.

(1) Determine the intervals on which a function is increasing / decreasing.



Increasing on (a,b)

f is increasing on (a,b) if $f(x_1) \le f(x_2)$ whenever $x_1 < x_2$.



Decreasing on (a,b)

$f(x_1) \geq f(x_2)$ whenever $x_1 < x_2$

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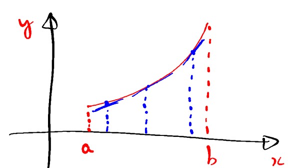
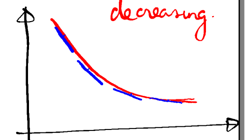
Relation between the first derivative and increasing / decreasing.



If $f'(x) > 0$ for every x in (a,b), then f is increasing on (a,b).



If $f'(x) < 0$ for every x in (a,b), then f is decreasing on (a,b)

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Strategy for finding the intervals of increasing / decreasing of a function.

Step 1: ~~Find all the critical #s of f.~~ (places where $f' = 0$ or f' is undefined.)

Step 2: $c_1 \quad c_2 \quad c_3 \quad c_4$ Consider the sign of f' on each interval. (the critical #s divide the number line into intervals)

Step 3: If $f' > 0$ on an interval, f is increasing on that interval. If $f' < 0$ on an interval, f is decreasing on that interval

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E.g. $f(x) = -x^3 + 12x$. Determine the intervals of increasing / decreasing of f.

Step 1: $f'(x) = \boxed{-3x^2 + 12}$.

$f'(x) = 0$; $-3x^2 + 12 = 0$; $-3x^2 = -12$

$x^2 = 4$

$x = 2; -2$

Step 2:

x	-3	-2		2	
$f'(x)$	$-$	0	$+$	0	$-$
$f(x)$	↘	rel. min	↗	rel. max	↘

Step 3: Conclusion.

f is increasing on $(-2, 2)$

f is decreasing on $(-\infty, -2) \cup (2, \infty)$

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E.g. $f(x) = x + \frac{9}{x}$. Intervals of increasing / decreasing for f.

$f'(x) = 1 - \frac{9}{x^2}$

* Even though f' is undefined at $x = 0$, $x = 0$ is NOT a critical # b/c the original function isn't defined at 0.

* $1 - \frac{9}{x^2} = 0$. $\frac{9}{x^2} = 1$; $x^2 = 9$; $x = \pm 3$

x	-4	-3	-1	0	1	3	4
$f'(x)$	$+$	0	$-$		$-$	0	$+$
$f(x)$	↗		↘		↘		↗

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Conclusion: f is increasing on $(-\infty, -3) \cup (3, \infty)$

f is decreasing on $(-3, 0) \cup (0, 3)$

E.g. $f(x) = \sin^2 x + \sin x$. on $\boxed{(0, 2\pi)}$

Increasing / Decreasing ?

$f'(x) = 2\sin x \cdot \cos x + \cos x = 0$

$\cos x \cdot (2\sin x + 1) = 0$

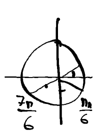
Either $\cos x = 0$ or $2\sin x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$.



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x	$\pi/2$	$7\pi/6$	$3\pi/2$	$11\pi/6$
$f'(x)$	0	0	0	0
$f(x)$				

Calculator needed.

The first derivative test. ↗ ↘

(1) If f changes from being increasing to being decreasing (f' changes from being $+$ to being $-$) at a point c, then c corresponds to a rel. ~~min~~ max

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(2) f' changes from being $-$ to being $+$ at a point c, then c corresponds to a relative min.

E.g. of using 1st derivative test to find relative extrema.

$$f(x) = \frac{x^2 - 2x + 1}{x+1}.$$

Use 1st derivative test to find rel. extrema of f.

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$$f(x) = \frac{x^2-2x+1}{x+1}$$

$$f'(x) = \frac{(2x-2)(x+1) - (x^2-2x+1)\cdot 1}{(x+1)^2}$$

$$= \frac{2x^2 \cancel{+2x} \cancel{-2x} - 2 - x^2 + 2x - 1}{(x+1)^2}$$

$$f'(x) = \frac{x^2+2x-3}{(x+1)^2}$$

f' undefined $\boxed{x=-1}$ — f is undefined

$f'=0$ if $x^2+2x-3=0$

$(x+3)(x-1)=0$

$\boxed{x=-3}$; $\boxed{x=1}$

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x		-3		-1		1	
$f'(x)$	$+$	0	$-$		$-$	0	$+$
$f(x)$	↗		↘		↘		↗

$$\frac{x^2+2x-3}{(x+1)^2}$$

* f has a relative max at $x=-3$.

$$f(-3) = \frac{(-3)^2 - 2(-3)+1}{-3+1} = -8$$

* f has relative min at $x=1$

$$f(1) = \frac{1^2 - 2\cdot 1 + 1}{1+1} = \boxed{0}$$

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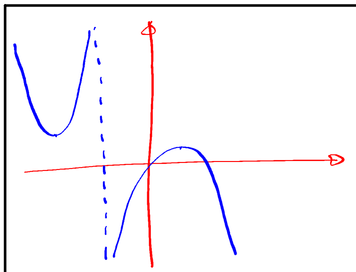

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$$f(x) = \frac{\sin x}{1+\cos^2 x}; \ (0, 2\pi)$$

Find relative extrema using 1st derivative.

$$f'(x) = \frac{\cos x\cdot(1+\cos^2 x) - \sin x\cdot(2\cos x\cdot(-\sin x))}{(1+\cos^2 x)^2}$$

$$= \frac{\cos x + \cos^3 x + 2\cos x \sin^2 x}{(1+\cos^2 x)^2}$$

$$= \frac{\cos x\,(1 + \cos^2 x + 2\sin^2 x)}{(1+\cos^2 x)^2}$$

$$= \frac{\cos x\,(1 + \underbrace{\cos^2 x + \sin^2 x}_{=1} + \sin^2 x)}{(1+\cos^2 x)^2}$$

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$$f' = \frac{-\cos x\,(2+\sin^2 x)}{(1+\cos^2 x)^2}$$

$$1 + \cos^2 x = 0$$

$\cos^2 x \geq 0$

f' is always defined.

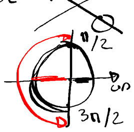
$$f' = 0$$

$$\cos x\,(2+\sin^2 x) = 0$$

$\cos x = 0$ or $\cancel{2+\sin^2 x = 0}$

on $(0, 2\pi)$

$x = \frac{\pi}{2}\ ;\ \frac{3\pi}{2}$



x	0		$\pi/2$		$3\pi/2$		2π
f'		$+$	0	$-$	0	$+$	
f		↗	Rel max	↘	Rel min	↗	

$= \cos x\,($