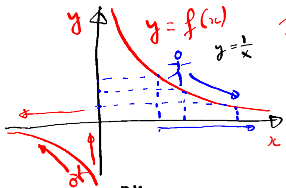
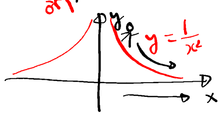
3.5 and 3.6. Limits at infinity and Curve Sketching

$f(x)$; $\lim_{x\to\infty} f(x)$; $\lim_{x\to -\infty} f(x)$?



$\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to -\infty} f(x) = 0$



$\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to -\infty} f(x) = 0$

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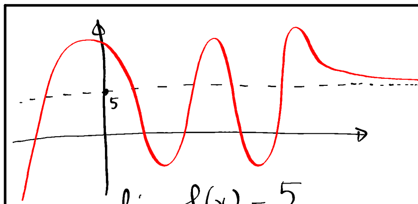

$\lim_{x\to\infty} f(x) = 5$

* Key observation:

$\lim_{x\to\infty} \frac{1}{x} = 0$; $\lim_{x\to-\infty} \frac{1}{x} = 0$

$\lim_{x\to\infty} \frac{1}{x^{17}} = 0$; $\lim_{x\to-\infty} \frac{1}{x^{17}} = 0$

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$\lim_{x\to\infty} \frac{1}{x^{1/2}} = \lim_{x\to\infty} \frac{1}{\sqrt{x}} = 0$

$\lim_{x\to-\infty} \frac{1}{x^{1/3}} = 0$

In general $\lim_{x\to\infty} \frac{1}{x^{n}} = 0$

$\lim_{x\to-\infty} \frac{1}{x^{n}} = 0$ positive rational #

E.g. $\lim_{x\to\infty} \frac{x^5+2}{x^6+3} \approx \frac{x^5}{x^6} = \frac{1}{x} \xrightarrow{x\to\infty} 0$

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$\lim_{x\to\infty} \frac{x^5+2}{x^6+3} = \lim_{x\to\infty} \frac{(x^5+2)/x^6}{(x^6+3)/x^6}$

$= \lim_{x\to\infty} \frac{\frac{1}{x} + \frac{2}{x^6}}{1 + \frac{3}{x^6}} = \frac{0}{1+0} = \frac{0}{1} = 0$

$\lim_{x\to\infty} \frac{x^5+1}{3x^5+7} = ?$ $\quad \frac{\cancel{x^5}}{3\cancel{x^5}} = \frac{1}{3}$ $\quad \frac{x^5}{x^4} = x \to \infty$

$\lim_{x\to\infty} \frac{(x^5+1)/x^5}{(3x^5+7)/x^5} = \lim_{x\to\infty} \frac{1+\frac{1}{x^5}}{3+\frac{7}{x^5}} = \frac{1}{3}$

$\lim_{x\to\infty} \frac{(x^5+6)/x^5}{(10x^4+11)/x^5} = \lim_{x\to\infty} \frac{1+6/x^5}{10/x + 11/x^5} = \frac{1}{0+0} = \infty$

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$\lim_{x\to\infty} \frac{x^5+2}{x^6+3} = 0$

$\lim_{x\to\infty} \frac{x^5+1}{3x^5+7} = \frac{1}{3}$

$\lim_{x\to\infty} \frac{x^6+6}{10x^4+11} = \infty$

$f(x) = \frac{a_n x^n + \cdots + a_1 x + a_0}{b_m x^m + \cdots + b_1 x + b_0}$

If $n < m$, $\lim_{x\to\infty \atop x\to-\infty} f(x) = 0$

If $n = m$, $\lim_{x\to\infty \atop x\to-\infty} f(x) = \frac{a_n}{b_m}$

If $n > m$, $\lim_{x\to\infty \atop x\to-\infty} f(x) = \infty$ $\quad \frac{x^6+6}{10x^4+11}$

E.g. $\lim_{x\to-\infty} \frac{2x^4 - x^3 + 10x^2 - x + 11}{-5x^4 + 7x^3 - 100x + 27} = -\frac{2}{5}$

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E.g. with functions with radical $\quad \lim_{x\to\pm\infty} \frac{1}{x^n} = 0$, $n > 0$

$\lim_{x\to\infty} \frac{\sqrt{x^2-1}}{2x-1}$ $\quad \sqrt{(-3)^2} = 3$ $\quad \boxed{\sqrt{x^2} = |x|}$

$= \lim_{x\to\infty} \frac{\sqrt{x^2\left(1-\frac{1}{x^2}\right)}}{x\left(2-\frac{1}{x}\right)} = \lim_{x\to\infty} \frac{\sqrt{x^2}\cdot\sqrt{1-\frac{1}{x^2}}}{x\cdot\left(2-\frac{1}{x}\right)}$

$= \lim_{x\to\infty} \frac{|x|\cdot\sqrt{1-\frac{1}{x^2}}}{x\cdot\left(2-\frac{1}{x}\right)} = \lim_{x\to\infty} \frac{\cancel{x}\cdot\left(\sqrt{1-\frac{1}{x^2}}\right)}{\cancel{x}\cdot\left(2-\frac{1}{x}\right)}$

means $x > 0$

$= \lim_{x\to\infty} \frac{\sqrt{1-\frac{1}{x^2}}}{2-\frac{1}{x}} = \frac{\sqrt{1}}{2} = \frac{1}{2}.$

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$$\lim_{x\to-\infty} \frac{\sqrt{x^2-1}}{2x-1} = \lim_{x\to-\infty}\frac{\sqrt{x^2\left(1-\frac{1}{x^2}\right)}}{x\left(2-\frac{1}{x}\right)} \qquad \boxed{x}$$

$$= \lim_{x\to-\infty} \frac{\sqrt{x^2}\sqrt{\left(1-\frac{1}{x^2}\right)}}{x\cdot\left(2-\frac{1}{x}\right)} = \lim_{x\to-\infty}\frac{|x|\sqrt{1-\frac{1}{x^2}}}{x\cdot\left(2-\frac{1}{x}\right)}$$

$$= \lim_{x\to-\infty}\frac{-\cancel{x}\sqrt{1-\frac{1}{x^2}}}{\cancel{x}\cdot\left(2-\frac{1}{x}\right)} = \lim_{x\to-\infty}\frac{-\sqrt{1-\frac{1}{x^2}}}{2-\frac{1}{x}} = -\frac{1}{2}$$

$$\boxed{\overset{-3}{|x|} = \begin{cases} x & \text{if } x>0 \\ \overset{-(-3)}{-x} & \text{if } x<0 \end{cases}} \quad |-3| = 3 = -(-3)$$

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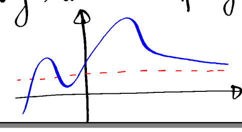
Eg $\lim_{x\to\infty}\frac{\sin x}{x}$ Squeeze Theorem

$$-\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x} \qquad \lim_{x\to\infty}\frac{\sin x}{x} = 0$$

As $x\to\infty$ $\quad 0 \quad$ $x\to\infty$

* Using limits at infinity to find H.A.



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We say that the horizontal line $y = a$ is a H.A. of $f(x)$ if either $\boxed{\lim_{x\to\infty} f(x) = a}$ or $\boxed{\lim_{x\to-\infty} f(x) = a}$

What is the horizontal asymptote of the function $f(x) = \dfrac{4x^5 - x^3 + x^2 + 1}{2x^5 - 10x^3 - 11x + 7}$

H.A. $\boxed{y = 2}$

H.A. of $f(x) = \dfrac{3x}{x^2+9}$: $y=0$ is a H.A

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