3.7 – Optimization Problems

Maximize / Minimize some quantity.

Find a function $f(x)$ which represents that quantity.
- Find max/min over close interval
- First derivative test
- Second derivative test.

E.g.1 60 ft of fencing. Enclose a rectangular area as large as possible.

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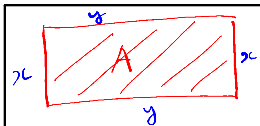


Find width & length of the rectangle that give max area.

max A. $2x+2y=60$

$\boxed{x+y=30}$ $y=30-x$

x	y	Area
20	10	200 ft^2
25	5	125 ft^2
29.1	0.9	→ area

Find x, y such that $\boxed{A=xy}$ is maximum.

$A=x(30-x)$. $A(x)=x\cdot(30-x)$

Maximize?

Aug 1-12:09 PM

$A(x)=x\cdot(30-x)=30x-x^2$

$A'(x)=30-2x=0$. This implies that $x=15$

x	0	1	15	16	30
$A'(x)$		+	0	−	
$A(x)$		↗	max	↘	

$x+y=30$

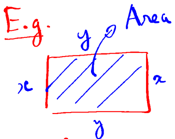
$y=15$

$A(x)$ is maximum when $x=15$.

The dimensions of the rectangle with max area are $x=y=15$. The max area $=15\cdot 15=225$ ft^2

Aug 1-12:13 PM

E.g.



Area $=40$ ft^2.

$\boxed{xy=40}$; $y=\frac{40}{x}$

Perimeter $=\boxed{2x+2y}$

enclose an area of 40 ft^2

Find dimensions of rectangle that will minimize the amount of fencing used.

$P(x)=2x+2\cdot\frac{40}{x}=2x+\frac{80}{x}$.

$P'(x)=2-\frac{80}{x^2}=0$

$2=\frac{80}{x^2}$. $x^2=\frac{80}{2}=40$. $\boxed{x=\sqrt{40}}$

Aug 1-12:19 PM

x	0	1	$\sqrt{40}$	8	∞
$P'(x)$		−	0	+	
$P(x)$		↘	min	↗	

$P'(x)=2-\frac{80}{x^2}$

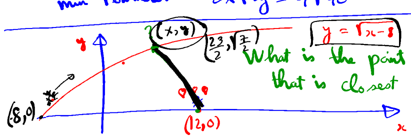
$P(x)$ is min when $x=\sqrt{40}$.

$xy=40$

$y=\frac{40}{x}=\frac{40}{\sqrt{40}}=\sqrt{40}$

The dimensions that will minimize P is $x=y=\sqrt{40}$.

min Perimeter is $2x+2y=4\sqrt{40}$



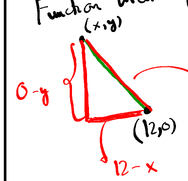
What is the point on the red curve that is closest to (12, 0)?

Aug 1-12:26 PM

Find (x,y) on the curve such that the distance from (x,y) to $(12,0)$ is minimum.

Function that represents that distance?

distance $=\cancel{\frac{0-y}{12-x}}$



distance $=\sqrt{(12-x)^2+(0-y)^2}$

distance $=\sqrt{(12-x)^2+y^2}$.

distance formula: (x_1,y_1) $\boxed{\text{distance}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$ (x_2,y_2)

Aug 1-12:28 PM

min. $\sqrt{(12-x)^2 + y^2}$ $\qquad y = \sqrt{x-8}$

$= \sqrt{(12-x)^2 + (\sqrt{x-8})^2} = \boxed{\sqrt{(12-x)^2 + x-8}}$

min

Useful trick: Instead minimizing $\sqrt{(12-x)^2 + x-8}$, we can just minimize $(12-x)^2 + x - 8$ and when we get the answer we just take the square root of it.

min. $f(x) = (12-x)^2 + x - 8$

$f'(x) = 2(12-x)\cdot(-1) + 1$

$= -24 + 2x + 1 = 2x - 23 = 0 \Rightarrow x = \frac{23}{2}$.

0

Aug 1-12:38 PM

x	-8	0	$\frac{23}{2}$		∞
$f'(x)$		$-$	0	$+$	
$f(x)$		$\searrow$	min	$\nearrow$	

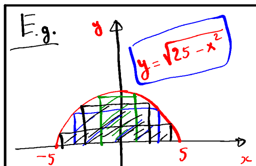
distance is min. when $x = \frac{23}{2}$. $y = \sqrt{\frac{23}{2} - 8} = \sqrt{\frac{23}{2} - \frac{16}{2}}$

$= \sqrt{\frac{7}{2}}$.

$\boxed{\left(\frac{23}{2}, \sqrt{\frac{7}{2}}\right)}$

Aug 1-12:44 PM

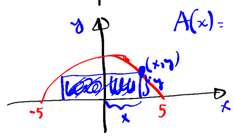
E.g.



Find the length & width of the rectangle inscribed in this semicircle of maximum area.

$2xy \leftarrow$ Area.

$A(x) = 2x\sqrt{25-x^2} \leftarrow$ maximize.



$A'(x) = 2\cdot\left(\sqrt{25-x^2} + x\cdot\frac{1}{2}\cdot(25-x^2)^{-\frac{1}{2}}\cdot(-2x)\right)$

$= 2\cdot\left(\sqrt{25-x^2} + \frac{x\cdot(-\cancel{2}x)}{\cancel{2}\sqrt{25-x^2}}\right) = 2\cdot\left(\sqrt{25-x^2} - \frac{x^2}{\sqrt{25-x^2}}\right)$

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Aug 1-12:54 PM