4.1. Antiderivatives.

Definition: f defined on an interval I.
An antiderivative of f is a function F such that $F'(x) = f(x)$ for every x in I.

E.g. $f(x) = x^2$. Antiderivative? $\frac{1}{3}x^3$

$$\left(\frac{1}{3}x^3\right)' = \frac{1}{3}\cdot 3x^2 = x^2$$

$\frac{1}{3}x^3 + 1000$; $\boxed{\frac{1}{3}x^3 - 2017}$ A general antiderivative $\frac{1}{3}x^3 + C$; C: any constant

Notation: $\boxed{\int x^2\,dx} = \boxed{\frac{1}{3}x^3 + C}$

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$\int x\,dx \overset{?}{=} \frac{1}{2}x^2 + C$ $\int x^4 dx = \frac{1}{5}x^5 + C.$

$\int x^3\,dx = \frac{1}{4}x^4 + C$ $\int x^n\,dx = \boxed{\frac{1}{n+1}x^{n+1} + C.}$

n: any number

$n=-1$ $\boxed{\int x^{-1}dx} = \frac{1}{-1+1}x^{-1+1} + C$

$= \frac{1}{0}$ problem!

Formula: $\boxed{\int x^n dx = \boxed{\frac{x^{n+1}}{n+1}} + C \text{ for } n \neq -1}$

integrand

$\left(-\frac{1}{x}\right)' = (-x^{-1})'$
$= -(-1\,x^{-2}) = \frac{1}{x^2}$ take derivative

$\int x^{\pi}\,dx = \frac{x^{\pi+1}}{\pi+1} + C$

$\int \frac{1}{x^2}dx \overset{?}{=} \int x^{-2}dx = \frac{x^{-2+1}}{-2+1} + C$

$= \frac{x^{-1}}{-1} + C = \boxed{-\frac{1}{x} + C}$ check

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$$\int \sqrt{x}\,dx \overset{?}{=} \int x^{1/2}dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{x^{3/2}}{\frac{3}{2}} + C$$
$$= \frac{2}{3}x^{3/2} + C.$$

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C. \quad (n \neq -1)}$$

$\int \boxed{7}x^2\,dx = 7\frac{x^3}{3} + C$

$= 7\cdot\boxed{\int x^2 dx}$

$$\boxed{\int k\cdot f(x)\,dx = k\cdot \int f(x)\,dx} \quad k: \text{constant}$$

We can always pull the constant out of the $\int$

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$$\int (x^2 + x^3)\,dx = \frac{x^3}{3} + \frac{x^4}{4} + C.$$

$$\boxed{\int (f(x) \pm g(x))\,dx = \int f(x)dx \pm \int g(x)dx}$$

$\cancel{\int f(x)\cdot g(x)\,dx} = \cancel{\int f(x)dx \int g(x)dx}$

$\int x^2\cdot x^3$
$\int x^5 = \frac{x^6}{6} + C$

$\int x^2 \cdot \int x^3$
$\frac{x^3}{3}\cdot\frac{x^4}{4}$
$\cancel{\frac{x^7}{12}}$

$\cancel{\int \frac{f(x)}{g(x)}dx = \frac{\int f(x)dx}{\int g(x)dx}}$

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Ex: (1) $\int \left(\sqrt[3]{x} - \frac{1}{x^3}\right)dx \overset{?}{=}$

(2) $\int \left(x^{-1/4} + \frac{3}{\sqrt[3]{x}}\right)dx \overset{?}{=}$

(3) $\int \frac{2x^5 - x^4 + x^3 + 1}{x^2}\,dx \overset{?}{=}$

(4) $\int (x-1)(2x+3)\,dx \overset{?}{=}$

$\int k\,dx = kx + C$ derivative

$$\int\left(\sqrt[3]{x} - \frac{1}{x^3}\right)dx = \int\left(\boxed{x^{1/3}} - x^{-3}\right)dx = \frac{3x^{4/3}}{4} - \frac{x^{-2}}{-2}$$
$$= \frac{3}{4}x^{4/3} + \frac{1}{2x^2} + C.$$

$\int \boxed{2^{\pi}}\,dx = 2^{\pi}x + C.$

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(2) $\int\left(x^{-1/4} + \frac{3}{\sqrt[3]{x}}\right)dx = \frac{4}{3}x^{3/4} + \frac{9}{4}x^{4/3} + C$

6 $\frac{9}{2}x^{2/3} + C$

$\int\left(x^{-1/4} + 3x^{-1/3}\right)dx$

(3) $\int \frac{2x^5 - x^4 + x^3 + 1}{x^2}\,dx$

$$= \int\left(2x^3 - x^2 + x + \frac{1}{x^2}\right)dx$$
$$= 2\cdot\frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} - x^{-1} + C = \frac{x^4}{2} - \frac{x^3}{3} + \frac{x^2}{2} - \frac{1}{x} + C.$$

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$$\int \boxed{(x-1)(2x+3)}\,dx$$
$$= \int (2x^2+3x-2x-3)\,dx = \int (2x^2+x-3)\,dx$$
$$= \frac{2}{3}x^3 + \frac{x^2}{2} - 3x + C.$$

Basic antiderivatives

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$
$$\int \sec^2 x\,dx = \tan x + C. \qquad \int \sec x \tan x\,dx = \sec x + C$$
$$\int \csc^2 x\,dx = -\cot x + C. \qquad \int \csc x \tan x\,dx = -\csc x + C$$

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Eg. let $\boxed{f(x) = x^3 - x + 1.}$

Find the function $F(x)$ such that $\boxed{F'(x) = f(x)}$ and $\boxed{F(0) = 6.}$

$$F(x) = \int (x^3 - x + 1)\,dx = \boxed{\frac{x^4}{4} - \frac{x^2}{2} + x + C.}$$
$$F(0) = C = 6$$
$$\boxed{F(x) = \frac{x^4}{4} - \frac{x^2}{2} + x + 6.}$$

Eg. Find the function $y = f(x)$ such that $\boxed{y(0) = 2, \; y(1) = -1.}$

and $\boxed{\frac{d^2y}{dx^2}} = \sqrt{x} - 2x.$

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$$\int \boxed{(\sqrt{x} - 2x)}\,dx = \int (x^{1/2} - 2x)\,dx = \frac{2}{3}x^{3/2} - x^2 + C.$$
$$\int \left(\frac{2}{3}x^{3/2} - x^2 + C\right)dx = \boxed{\frac{2}{3}\cdot\frac{2}{5}\cdot x^{5/2} - \frac{x^3}{3} + Cx + D}$$

$y(0) = 2 \; ; \; y(1) = -1$

$y(0) = 2. \implies \boxed{D = 2}$

$$\boxed{y = \frac{4}{15}x^{5/2} - \frac{x^3}{3} + \frac{44}{15}x + 2}$$

$$y(1) = -1 \implies \frac{2}{3}\cdot\frac{2}{5}\cdot 1 - \frac{1}{3} + C\cdot 1 + 2 = -1$$
$$\implies \frac{4}{15} - \frac{1}{3} + C = -3$$
$$\implies \frac{1}{15} + C = 3 \implies C = 3 - \frac{1}{15} = \boxed{\frac{44}{15}}$$

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