Find Definite integrals by substitution.

E.g. $\int_0^1 x\sqrt{1-x^2}\,dx = -\frac{1}{2}\int_0^1 \sqrt{1-x^2}\,(-2x\,dx) = \boxed{-\frac{1}{2}\int_1^0 \sqrt{u}\,du}$

let $u = 1-x^2$ $du = -2x\,dx$

1st way to proceed: $x=0$ plug in $1-x^2 \to u=1$; $x=1$ plug in $1-x^2 \to u=0$ } new bounds for integral

$-\frac{1}{2}\int_1^0 \sqrt{u}\,du = \frac{1}{2}\int_0^1 \sqrt{u}\,du = \frac{1}{2}\cdot\frac{2u^{3/2}}{3}\Big|_0^1 = \frac{u^{3/2}}{3}\Big|_0^1 = \frac{1}{3} - 0 = \boxed{\frac{1}{3}}$

2nd way to proceed

$-\frac{1}{2}\int \sqrt{u}\,du = -\frac{1}{2}\cdot\frac{2u^{3/2}}{3} = -\frac{u^{3/2}}{3} = -\frac{(1-x^2)^{3/2}}{3}\Big|_0^1 = -\left[0 - \frac{1}{3}\right] = \boxed{\frac{1}{3}}$

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E.g. $\frac{1}{2}\int_1^5 \frac{x}{\sqrt{2x-1}}\,2\cdot dx$ let $u = 2x-1$. $x = \frac{u+1}{2}$. $du = 2\,dx$

$x=1 \to u = 2\cdot 1 - 1 = 1$.
$x=5 \to u = 2\cdot 5 - 1 = 9$

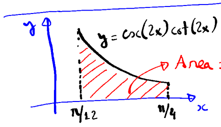
$\frac{1}{2}\int_1^9 \frac{u+1}{2\sqrt{u}}\,du$

$= \frac{1}{4}\int_1^9 \frac{u+1}{\sqrt{u}}\,du = \frac{1}{4}\int_1^9 (u+1)\cdot u^{-1/2}\,du$

$= \frac{1}{4}\int_1^9 \left(u^{1/2} + u^{-1/2}\right)du = \frac{1}{4}\cdot\left(\frac{2u^{3/2}}{3} + 2u^{1/2}\right)\Big|_1^9$

$= \frac{1}{4}\left[\left(\frac{2}{3}(9)^{3/2} + 2\cdot(9)^{1/2}\right) - \left(\frac{2}{3}+2\right)\right]$ $(\sqrt{9})^3$

$\frac{1}{4}\left[\left(18+6\right) - \left(\frac{8}{3}\right)\right] = \frac{1}{4}\cdot\frac{64}{3} = \boxed{\frac{16}{3}}$ $24 - \frac{8}{3} =$



$\frac{1}{2}\int_{\pi/12}^{\pi/4} \csc(2x)\cot(2x)\,2dx$ let $u = 2x$; $du = 2dx$

$\frac{1}{2}\int_{\pi/6}^{\pi/2} \csc(u)\cot(u)\,du$

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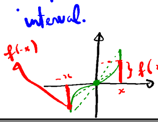
$\frac{1}{2}\int_{\pi/6}^{\pi/2} \csc(u)\cot(u)\,du = -\frac{1}{2}\csc(u)\Big|_{\pi/6}^{\pi/2}$

$= -\frac{1}{2}\left[\csc\left(\frac{\pi}{2}\right) - \csc\left(\frac{\pi}{6}\right)\right]$

$= -\frac{1}{2}\cdot\left[1-2\right] = -\frac{1}{2}\cdot(-1) = \boxed{\frac{1}{2}}$

* A useful trick:
Definite integrals of even and odd functions over symmetric interval.

odd function: { whose graph is symmetric with respect to the origin. algebraic: $\boxed{f(-x) = -f(x)}$



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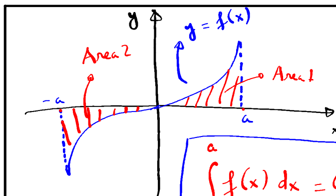
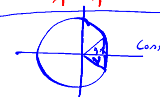


$\int_{-a}^{a} f(x)\,dx = $ Area 1 − Area 2 $= 0.$

f is odd

$\boxed{\int_{-a}^{a} f(x)\,dx = 0 \text{ if } f \text{ is odd.}}$

E.g. $\int_{-5}^{5} \boxed{x\sqrt{x^2+\cos x}}\,dx = 0$
$f(x)$



$f(-x) = (-x)\cdot\sqrt{(-x)^2 + \cos(-x)}$
$= \boxed{-x\cdot\sqrt{x^2+\cos x}} = -f(x) \Rightarrow f$ is odd.

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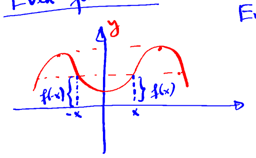
$\int_{-\pi/4}^{\pi/4} \frac{\sin x}{\sqrt{x^4+x^2}}\,dx = 0$
is an odd function.
$f(x)$

$f(-x) = \frac{\sin(-x)}{\sqrt{(-x)^4 + (-x)^2}} = \frac{-\sin x}{\sqrt{x^4+x^2}} = -f(x)$

Even functions

Even function: { graph is symmetric w.r.t. y-axis; algebraically: $f(-x) = f(x)$.



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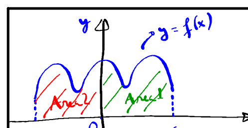

$\int_{-a}^{a} f(x)\,dx = $ Area 1 + Area 2
$= 2\cdot$ Area 1
$= 2\cdot\int_0^a f(x)\,dx$

$\boxed{\int_{-a}^{a} f(x)\,dx = 2\cdot\int_0^a f(x)\,dx}$ for even functions.

E.g. $\int_{-\pi/2}^{\pi/2} \boxed{\sin^2 x\cos x}\,dx = \boxed{2\cdot\int_0^{\pi/2} \sin^2 x\cos x\,dx}$
$f(x)$

$f(-x) = \sin^2(-x)\cdot\cos(-x)$
$= (-\sin x)^2\cdot\cos x = \sin^2 x\cos x = f(x) \Rightarrow f$ is even.

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$$2\cdot\int_0^{\pi/2} \sin^2 x\cdot \underbrace{\cos x\,dx}_{du}$$

Let $u = \sin x$

$du = \cos x\,dx$

$x = 0;\ u = \sin 0 = 0$

$x = \frac{\pi}{2};\ u = \sin\frac{\pi}{2} = 1$

$$2\cdot\int_0^1 u^2\cdot du = 2\cdot\frac{u^3}{3}\Big|_0^1 = 2\cdot\left(\frac{1}{3}-\frac{0}{3}\right)$$

$$= \boxed{\frac{2}{3}}.$$

f: odd function. $\int_{-a}^{a} f(x)\,dx = 0$

f: even function $\int_{-a}^{a} f(x)\,dx = 2\cdot\int_0^a f(x)\,dx.$

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