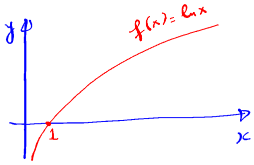
5.1. (1) Derivative of Logarithmic functions
(2) Logarithmic Differentiation

(1) $f(x) = \ln x$

$\ln(1) = 0$; $\ln(e) = 1$; $\ln(e^2) = 2$

$\ln(e^x) = x$. $\ln(0)$ undefined.



$\ln(ab) = \ln a + \ln b$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$.

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$\boxed{(\ln x)' = \frac{1}{x}}$; $\frac{d}{dx}(\ln x) = \frac{1}{x}$.

$\boxed{u: \text{function of } x}$

$(\ln(u))' = \frac{1}{u} \cdot u' = \frac{u'}{u}$

$\boxed{(\ln u)' = \frac{u'}{u}}$; $\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$

E.g. $f(x) = \ln(\boxed{2x-1})$

$f'(x) = \frac{2}{2x-1}$

$f(x) = \boxed{x^2} \cdot \boxed{\ln(\boxed{2x^2+1})}$

$f'(x) = 2x \cdot \ln(2x^2+1) + x^2 \cdot \frac{4x}{2x^2+1}$

$= 2x \cdot \ln(2x^2+1) + \frac{4x^3}{2x^2+1}$.

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$\left(\ln\left(\sqrt{x^2-4}\right)\right)' = ?$ $\ln(a^b) = b\ln(a)$

$\left(\boxed{\ln(x^2-4)^{\frac{1}{2}}}\right)' = \left(\frac{1}{2} \cdot \ln(x^2-4)\right)' = \frac{1}{2} \cdot \left(\ln(x^2-4)\right)'$

$= \frac{1}{2} \cdot \frac{2x}{x^2-4} = \frac{x}{x^2-4}$ $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$\left(\ln\left(\boxed{\sqrt{x^2-4}}\right)\right)' = \frac{\left(\sqrt{x^2-4}\right)'}{\sqrt{x^2-4}} = \frac{\frac{1}{2\sqrt{x^2-4}} \cdot 2x}{\sqrt{x^2-4}}$

$= \frac{\frac{x}{\sqrt{x^2-4}}}{\sqrt{x^2-4}} = \frac{x}{\sqrt{x^2-4}} \cdot \frac{1}{\sqrt{x^2-4}} = \frac{x}{x^2-4}$

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$\left(\ln\left(\boxed{x \cdot (x^2+3)^3}\right)\right)'$ $\left(\ln(a \cdot b) = \ln a + \ln b\right.$

$= \left(\ln x + \ln(x^2+3)^3\right)'$

$= \left(\ln x + 3\ln(x^2+3)\right)' = (\ln x)' + 3\left(\ln(x^2+3)\right)'$

$= \frac{1}{x} + 3 \cdot \frac{2x}{x^2+3} = \boxed{\frac{1}{x}} + \frac{6x}{x^2+3}$

Ex. Find the derivative of the given function.

(1) $f(x) = \ln\left(\frac{2x}{x+3}\right)$ (2) $g(x) = \ln(\ln x)$

(3) $h(t) = \ln\left(\sqrt[3]{\frac{t-1}{t+1}}\right)$

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Solution:

(1) $\left(\ln\left(\frac{2x}{x+3}\right)\right)' = \left(\ln(2x) - \ln(x+3)\right)'$

$= \left(\ln(\boxed{2x})\right)' - \left(\ln(x+3)\right)'$

$= \frac{2}{2x} - \frac{1}{x+3} = \boxed{\frac{1}{x} - \frac{1}{x+3}}$

(2) $\left(\ln(\boxed{\ln x})\right)' = \frac{\frac{1}{x}}{\ln x} = \frac{1}{x} \cdot \frac{1}{\ln x} = \boxed{\frac{1}{x \ln x}}$

$(\ln \boxed{u})' = \frac{u'}{u}$

(3) $\left(\ln\left(\sqrt[3]{\frac{t-1}{t+1}}\right)\right)' = \left(\ln\left(\frac{t-1}{t+1}\right)^{\frac{1}{3}}\right)' = \left(\frac{1}{3}\ln\left(\frac{t-1}{t+1}\right)\right)'$

$= \frac{1}{3} \cdot \left(\ln(t-1) - \ln(t+1)\right)'$

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$\frac{1}{3} \cdot \left(\frac{1}{t-1} - \frac{1}{t+1}\right)$. $\ln x$

Note: $(\ln|x|)' = \frac{1}{x}$.

E.g. $\left(\ln(\boxed{\csc x})\right)' = \frac{-\csc x \cot x}{\csc x} = -\cot x$.

$\boxed{\int \cot x \, dx} = -\ln(\csc x) + C$

$\left(\ln\left|\sec x + \tan x\right|\right)' = \frac{1}{\sec x + \tan x} \cdot \left(\sec x \tan x + \sec^2 x\right)$

$= \frac{\sec x \cancel{(\tan x + \sec x)}}{\cancel{\sec x + \tan x}} = \boxed{\sec x}$

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$$\int \sec x\,dx = \ln|\sec x + \tan x| + C.$$

$$(\ln x)' = \frac{1}{x}\ ;\quad (\ln u)' = \frac{u'}{u}\ ;\ u \text{ is a function of } x.$$

Implicit Differentiation.

$$\ln(xy) + 5x = 30\ ;\quad \frac{dy}{dx} = ?$$

$$\frac{d}{dx}\left(\ln(xy) + 5x\right) = \frac{d}{dx}(30)$$

$$\frac{d}{dx}\left(\ln(xy)\right) + \frac{d}{dx}(5x) = 0$$

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$$\frac{d}{dx}(\ln x + \ln y) + 5 = 0$$

$$\frac{1}{x} + \frac{1}{y}\cdot\frac{dy}{dx} + 5 = 0$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = -5 - \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{-5 - \frac{1}{x}}{\frac{1}{y}} = \frac{\frac{-5x-5}{x}}{\frac{1}{y}} =$$

$$\boxed{\frac{dy}{dx} = \frac{-5y(x+1)}{x}}$$

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Logarithmic Differentiation.

E.g. $\boxed{y = \sqrt{x^2(x+1)(x+2)}}$

$y' = ?$

Technique of log. diff.

$$\ln y = \ln\sqrt{x^2(x+1)(x+2)}\,. \quad (\text{Take } \ln \text{ of both sides})$$

$$\ln y = \ln\left(x^2(x+1)(x+2)\right)^{1/2}$$

$$\ln y = \frac{1}{2}\ln\left(x^2(x+1)(x+2)\right)$$

$$\ln y = \frac{1}{2}\left[\ln x^2 + \ln(x+1) + \ln(x+2)\right]$$

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$$\ln y = \frac{1}{2}\left[\boxed{2\ln x} + \boxed{\ln(x+1)} + \boxed{\ln(x+2)}\right]$$

Take the derivative of both sides with respect to x.

$$\frac{y'}{y} = \frac{1}{2}\left[\frac{2}{x} + \frac{1}{x+1} + \frac{1}{x+2}\right]$$

Multiply both sides by y to get y' by itself.

$$y' = \frac{1}{2}\left[\frac{2}{x} + \frac{1}{x+1} + \frac{1}{x+2}\right]\cdot y$$

$$\boxed{y' = \frac{1}{2}\left[\frac{2}{x} + \frac{1}{x+1} + \frac{1}{x+2}\right]\cdot\sqrt{x^2(x+1)(x+2)}}$$

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E.g. $y = \dfrac{(x+1)(x+2)(x+3)}{(x-1)(x-2)(x-3)}$

Find y' ?

Log. diff. $\ln y = \boxed{\ln\dfrac{(x+1)(x+2)(x+3)}{(x-1)(x-2)(x-3)}}$

$$\ln y = \ln(x+1) + \ln(x+2) + \ln(x+3) - \ln(x-1) - \ln(x-2) - \ln(x-3)$$

$$\frac{y'}{y} = \underbrace{\frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3} - \frac{1}{x-1} - \frac{1}{x-2} - \frac{1}{x-3}}$$

$$y' = \left(\qquad\qquad\right)\cdot y$$

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