5.2 - Log Rule for Integration

$\boxed{\int x^n\,dx} = \frac{x^{n+1}}{n+1} + C.\quad n \neq -1.$

When $n=-1$. $\int \frac{1}{x}\,dx = \int \frac{dx}{x} = \ln|x| + C$

$\boxed{\int \frac{dx}{x} = \ln|x| + C}$

$\boxed{\int \frac{u'\,dx}{u} = \ln|u| + C.}$ $\boxed{\int \frac{du}{u} = \ln|u| + C}$

E.g. $\int \frac{dx}{x-5} \overset{?}{=} \ln|x-5| + C$

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$-\frac{1}{3}\int \frac{-3x^2\,dx}{5-x^3}$ let $u = 5 - x^3$. $\boxed{du = -3x^2\,dx}$

$-\frac{1}{3}\int \frac{du}{u} = -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|5-x^3| + C.$

E.g. $\frac{1}{3}\int \frac{3(x^2+4x)\,dx}{x^3+6x^2+5}$ let $u = x^3+6x^2+5$, $du = (3x^2+12x)\,dx$

$\frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|x^3+6x^2+5| + C.$

E.g. $\int \frac{1}{x\ln(x^3)}\,dx.$ let $u = \ln x$; $du = \frac{1}{x}\,dx$

$\int \frac{1}{3x\ln(x)}\,dx = \int \frac{du}{3u} = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u| + C$

$= \boxed{\frac{1}{3}\ln|\ln x| + C}$

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$\int \frac{dx}{1+\sqrt{x}}$ let $\boxed{u = 1 + \sqrt{x}}$

$du = \frac{1}{2\sqrt{x}}\,dx \Rightarrow dx = 2\boxed{\sqrt{x}}\,du$

$\boxed{dx = 2\cdot(u-1)\,du.}$

$\int \frac{dx}{u}$

$\int \frac{2(u-1)\,du}{u} = 2\int \frac{u-1}{u}\,du$

$= 2\int \left(1 - \frac{1}{u}\right)du$

$= 2\cdot(u - \ln|u|) + C$

$= \boxed{2\left(1+\sqrt{x} - \ln|1+\sqrt{x}|\right) + C}.$

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$\int \boxed{\frac{2x^2+7x-3}{x-3}}\,dx$

Use long division before we integrate.

$$\begin{array}{r} \boxed{2x+13} \\ \boxed{x}-3 \;\big)\; \boxed{2x^2} + 7x - 3 \\ -(2x^2 - 6x) \\ \boxed{13x} - 3 \\ -(13x - 39) \\ \boxed{42} \end{array}$$

$2x+13 \to$ quotient

$42 \to$ Remainder

$\frac{2x^2}{x}$ $\frac{13x}{x}$

$\frac{2x^2+7x-3}{x-3} = \boxed{2x+13} + \frac{\boxed{42}}{x-3}$ (quotient, R)

$\int \left((2x+13) + \frac{42}{x-3}\right)dx$

$= x^2 + 13x + 42\cdot\ln|x-3| + C.$

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Ex. $\int \frac{x^4+x-4}{x^2+2}\,dx.$

$$\begin{array}{r} \boxed{x^2-2} \\ \boxed{x^2}+2 \;\big)\; x^4 \qquad + x - 4 \\ -(x^4 + 2x^2) \\ \boxed{-2x^2} + x - 4 \\ -(-2x^2 \qquad - 4) \\ \boxed{x} \end{array}$$

$x^2-2 \to$ quotient

x Remainder.

$\frac{x^4}{x^2}$ $\frac{-2x^2}{x^2} = -2$

$\int \frac{x^4+x-4}{x^2+2} = \int \left(x^2 - 2 + \frac{x}{x^2+2}\right)dx$ let $u = x^2+2$, $du = 2x\,dx$

$= \frac{x^3}{3} - 2x + \frac{1}{2}\int \frac{2x}{x^2+2}\,dx$

$= \frac{x^3}{3} - 2x + \frac{1}{2}\ln|x^2+2| + C$

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Integrals of basic trig functions

$\int \sin x\,dx = -\cos x + C$ $\int \cos x\,dx = \sin x + C.$

$\int \tan x\,dx = -\int \frac{-\sin x\,dx}{\cos x}$ let $u = \cos x$, $du = -\sin x\,dx$

$= -\int \frac{du}{u} = -\ln|u| + C = -\ln|\cos x| + C.$

$\boxed{\int \tan x\,dx = -\ln|\cos x| + C}$

$\boxed{\int \cot x\,dx = \ln|\sin x| + C}$

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$$\int \sec x \, dx = \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} \, dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

Let $u = \sec x + \tan x$

$du = (\sec x \tan x + \sec^2 x) \, dx$

$$\int \frac{du}{u} = \ln|u| + C = \ln|\sec x + \tan x| + C.$$

$$\boxed{\int \sec x \, dx = \ln|\sec x + \tan x| + C}$$

$$\boxed{\int \csc x \, dx = -\ln|\csc x + \cot x| + C}$$

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$$\frac{1}{2}\int (\sec(2x) + \tan(2x)) \, 2dx = \frac{1}{2}\int (\sec u + \tan u) \, du$$

Let $u = 2x$; $du = 2 \, dx$

$$= \frac{1}{2}\left(\ln|\sec u + \tan u| - \ln|\cos u|\right)$$

$$= \frac{1}{2}\left(\ln|\sec(2x) + \tan(2x)| - \ln|\cos(2x)|\right) + C.$$

$-\int \frac{\csc^2 x}{\cot x} \, dx$. $u = \cot x$; $du = -\csc^2 x \, dx$

$$-\int \frac{du}{u} = -\ln|u| + C = -\ln|\cot x| + C.$$

Definite integrals examples

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$$\int_1^e \frac{(1 + \ln x)^2}{x} \, dx \overset{?}{=}$$

Let $u = 1 + \ln x$. $du = \frac{1}{x} \, dx$

$x = 1$; $u = 1 + \ln(1) = 1$

$x = e$; $u = 1 + \ln(e) = 2$

$$\int_1^2 u^2 \, du = \frac{u^3}{3}\Big|_1^2 = \frac{8}{3} - \frac{1}{3} = \boxed{\frac{7}{3}}.$$

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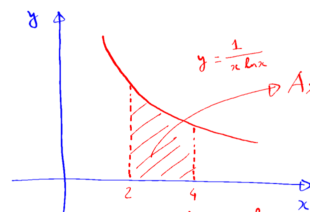

Let $u = \ln x$; $du = \frac{1}{x} \, dx$.

$x = 2$; $u = \ln 2$; $x = 4$; $u = \ln 4$

$$\int_2^4 \frac{1}{x \ln x} \, dx$$

$$\int_{\ln 2}^{\ln 4} \frac{1}{u} \, du = \ln|u| \Big|_{\ln 2}^{\ln 4} = \ln|\ln 4| - \ln|\ln 2|$$

$$= \ln\left|\frac{\ln 4}{\ln 2}\right| = \ln\left|\frac{2\cancel{\ln 2}}{\cancel{\ln 2}}\right|$$

$$= \boxed{\ln 2}$$

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