5.6. Inverse Trigonometric Functions – Differentiation

$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}(\operatorname{arccot} x) = \frac{-1}{1+x^2}$

$\frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}(\operatorname{arcsec} x) = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$ $\qquad$ $\frac{d}{dx}(\operatorname{arccsc} x) = \frac{-1}{|x|\sqrt{x^2-1}}$.

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$; $\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

$\frac{d}{dx}(\arcsin x)$ $\quad f(x) = \sin x$. $\boxed{g(x) = \boxed{\arcsin x}}$; $f'(x) = \cos x$

I.F.F. $g'(x) = \frac{1}{f'(g(x))}$ $\qquad f'(g(x)) = \cos(\arcsin x)$

$\boxed{g'(x)} = \frac{1}{\cos(\arcsin x)} = \frac{1}{\sqrt{1-x^2}}$

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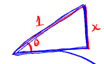
Simplify $\cos(\underbrace{\arcsin x}_{\theta}) = \cos\theta$



$\cos\theta = ?$ (in terms of x)

$\sin\theta = x$ $\qquad \sqrt{1-x^2}$

$\arcsin x = \theta$

$\cos\theta = \sqrt{1-x^2}$

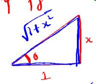
$\cos(\arcsin x) = \sqrt{1-x^2}$.

$\frac{d}{dx}(\arctan x)$; $f(x) = \tan x$; $f'(x) = \sec^2 x$

$g(x) = (\arctan x)$

$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{\boxed{\sec^2(\arctan x)}} = \frac{1}{1+x^2}$

Simplify $\sec^2(\underbrace{\boxed{\arctan x}}_{\theta})$ $\qquad \sec^2\theta$ (in terms of x)



$\tan\theta = x$ $\qquad \frac{1}{\cos^2\theta} = 1+x^2$

$\arctan x = \theta$

$\cos\theta = \frac{1}{\sqrt{1+x^2}}$; $\cos^2\theta = \frac{1}{1+x^2}$

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$(\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$ $\qquad (\operatorname{arccot} u)' = \frac{-u'}{1+u^2}$

$(\arccos u)' = \frac{-u'}{\sqrt{1-u^2}}$ $\qquad (\operatorname{arcsec} u)' = \frac{u'}{|u|\sqrt{u^2-1}}$

$\boxed{(\arctan u)' = \frac{u'}{1+u^2}}$ $\qquad (\operatorname{arccsc} u)' = \frac{-u'}{|u|\sqrt{u^2-1}}$.

E.g. $y = \arctan(\boxed{\sqrt{x}})$; $y' = \frac{\frac{1}{2\sqrt{x}}}{1+x} = \frac{1}{2\sqrt{x}\,(1+x)}$.

$y = \arcsin(x^2)$; $y' = \frac{2x}{\sqrt{1-x^4}}$.

E.g. Implicit Differentiation.

$\boxed{\arctan(xy) = \arctan(x+y)}$.

Find the equation of tangent line to this curve at $\boxed{(0,0)}$.

Slope $= \boxed{\frac{dy}{dx}}$ evaluated at $\boxed{(0,0)}$. $\qquad \frac{dy}{dx} \overset{?}{=} \frac{1-y}{x-1}$

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$\arctan(\boxed{xy}) = \arctan(x+y)$.

$\frac{1}{1+x^2y^2}\cdot\frac{d}{dx}(xy) = \frac{1}{1+(x+y)^2}\cdot\frac{d}{dx}(x+y)$.

$\frac{1}{1+x^2y^2}\cdot\left(y + x\frac{dy}{dx}\right) = \frac{1}{1+(x+y)^2}\cdot\left(1 + \frac{dy}{dx}\right)$

$\frac{y}{1+x^2y^2} + \frac{x}{1+x^2y^2}\frac{dy}{dx} = \frac{1}{1+(x+y)^2} + \frac{1}{1+(x+y)^2}\frac{dy}{dx}$

$x=0$; $y=0$.

$0 = 1 + \frac{dy}{dx}$. Hence, $\left.\frac{dy}{dx}\right|_{x=0;\,y=0} = \boxed{-1}$ Slope of tangent line

$\boxed{y = -x}$

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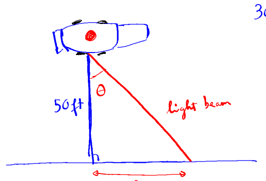

30 revolutions per minute

$30\cdot 2\pi = 60\pi$

$\boxed{\frac{d\theta}{dt} = 60\pi}$ per minute.

$\frac{dx}{dt} = ?$ $\quad \theta = 45°$

Write θ as a function of x.

$\tan\theta = \frac{x}{50}$

$\theta = \boxed{\arctan\left(\frac{x}{50}\right)}$

$\theta = 45°$

$\tan 45° = \frac{x}{50}$

$1 = \frac{x}{50}$, $x = 50$

$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{x}{50}\right)^2}\cdot\frac{1}{50}\cdot\frac{dx}{dt}$.

$\frac{d\theta}{dt} = \frac{1}{\left(1+\frac{x^2}{2500}\right)50}\frac{dx}{dt} \Longrightarrow \frac{d\theta}{dt} = \underbrace{\frac{1}{(1+1)\cdot 50}}_{100}\cdot\frac{dx}{dt}$

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$60\pi = \frac{1}{100}\frac{dx}{dt}$

$\frac{dx}{dt} = 6000\pi$.

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