

5.7. Integration with inverse trig functions

$a > 0$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$$

$$\frac{d}{du} \left(\arcsin \frac{u}{a} \right) = \frac{1}{\sqrt{1 - \left(\frac{u}{a}\right)^2}} \cdot \frac{1}{a} = \frac{1}{\sqrt{1 - \frac{u^2}{a^2}}} \cdot \frac{1}{a}$$

$$= \frac{1}{a \sqrt{1 - \frac{u^2}{a^2}}} = \frac{1}{\sqrt{a^2 - u^2}}$$

$$\int \frac{du}{a^2 + u^2} = \left(\frac{1}{a} \right) \arctan \frac{u}{a} + C$$

$$\frac{d}{du} \left(\arctan \frac{u}{a} \right) = \frac{1}{1 + \frac{u^2}{a^2}} \cdot \frac{1}{a} = \frac{1}{a + \frac{u^2}{a}} = \frac{1}{a^2 + u^2}$$

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$$\int \frac{du}{\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arccosh} \frac{|u|}{a} + C$$

$$\int \frac{dx}{(x^2+9)^{3/2}} = \int \frac{du}{u^2+9}$$

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \arcsin \frac{u}{a} + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$\int \frac{du}{u \sqrt{u^2 - a^2}} = \operatorname{arccsc} \frac{|u|}{a} + C$$

$$\int \frac{2}{\sqrt{x^2+9}} dx$$

E.g. 1 $\int \frac{dx}{\sqrt{1-4x^2}} = \frac{1}{2} \int \frac{2 dx}{1-(2x)^2}$ let $u=2x$, $du=2dx$

$$\frac{1}{2} \int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2} \arcsin u + C = \frac{1}{2} \arcsin(2x) + C$$

E.g. 2 $\int \frac{12}{1+9x^2} dx = \int \frac{12}{1+(3x)^2} dx$ let $u=3x$, $du=3dx$

$$\int \frac{4 du}{1+u^2} = 4 \arctan u + C = 4 \arctan(3x) + C$$

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E.g. 3 $\int \frac{1}{\sqrt{x^2-4}} dx$ let $u=x^2-4$, $du=2x dx$

$$\int \frac{du}{u \sqrt{u^2 - a^2}} = \operatorname{arccsc} \frac{|u|}{a} + C$$

$$\frac{1}{2} \int \frac{du}{u \sqrt{u^2 - 4}} = \frac{1}{2} \operatorname{arccsc} \frac{|u|}{2} + C$$

$$= \frac{1}{2} \operatorname{arccsc} \frac{|x^2|}{2} + C$$

E.g. 4 $\int \frac{x^2+3}{x \sqrt{x^2-4}} dx = \int \frac{x^2}{x \sqrt{x^2-4}} dx + \int \frac{3 dx}{x \sqrt{x^2-4}}$

$$\int \frac{x}{\sqrt{x^2-4}} dx = \sqrt{x^2-4} + 3 \operatorname{arccsc} \frac{|x|}{2} + C$$

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Method of completing the square $(x+a)^2 = x^2 + 2ax + a^2$

E.g. $\int \frac{dx}{x^2+4x+13} = \int \frac{dx}{(x^2+2(2)x+4)+9} = \int \frac{dx}{(x+2)^2+9}$

let $u=x+2$, $du=dx$

$$\int \frac{du}{u^2+9} = \frac{1}{3} \arctan \frac{u}{3} + C = \frac{1}{3} \arctan \frac{x+2}{3} + C$$

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$$\int \frac{2}{\sqrt{-x^2+4x}} dx = \int \frac{2}{\sqrt{-(x^2-4x)}} dx$$

$$= \int \frac{2 dx}{\sqrt{-(x^2-4x+4-4)}} = \int \frac{2 dx}{\sqrt{-(x-2)^2+4}}$$

$$= \int \frac{2 du}{\sqrt{4-u^2}} \rightarrow \int \frac{du}{\sqrt{4-u^2}} = \arcsin \frac{u}{2} + C$$

let $u=x-2$, $du=dx$

$$\int \frac{2 du}{\sqrt{4-u^2}} = 2 \left(\arcsin \frac{u}{2} \right) + C = 2 \arcsin \frac{x-2}{2} + C$$

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$$\int \frac{2x-5}{x^2+2x+2} dx = \int \frac{2x+2-2-5}{x^2+2x+2} dx = \int \frac{2x+2}{x^2+2x+2} dx - 5 \int \frac{dx}{x^2+2x+2}$$

let $u=x^2+2x+2$, $du=(2x+2) dx$

$$= \int \frac{du}{u} - 5 \int \frac{dx}{x^2+2x+2}$$

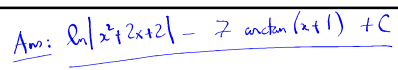
$$= \ln|u| - 5 \int \frac{dx}{(x+1)^2+1}$$

let $u=x^2+2x+2$, $du=(2x+2) dx$

$$\int \frac{du}{u} = \ln|u| + C = \ln|x^2+2x+2| + C$$

$$7 \int \frac{dx}{(x+1)^2+1} = 7 \int \frac{dx}{(t+1)^2+1} = 7 \arctan(t+1) + C = 7 \arctan(x+1) + C$$

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A rectangular box containing a handwritten mathematical expression in blue ink. The expression is $\ln|x^2+2x+2| - 7 \arctan(x+1) + C$. The word "Ans:" is written to the left of the expression, and the entire expression is underlined.

$$\text{Ans: } \ln|x^2+2x+2| - 7 \arctan(x+1) + C$$

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