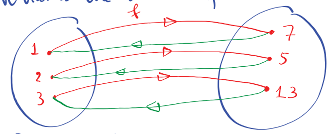
3.7 Inverse Functions

Goals: (1) Verify that 2 functions are inverse functions.

(2) Domain and Range of the inverse function of a function

(3) Find and evaluate inverse functions.

(1) What is the inverse function of a function f?



$f(1) = 7$, $f(2) = 5$, $f(3) = 13$ | $f^{-1}(7) = 1$, $f^{-1}(5) = 2$, $f^{-1}(13) = 3$

Function in green is called the inverse function of f. Notation for the inverse function is f^{-1}.

Domain of f / Range of f^{-1} ; Range of f / Domain of f^{-1}

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f^{-1} undoes what f does.

$f(x) = \frac{5}{9}(x-32)$ Convert Fahrenheit temperature to Celcius temperature

$x = 84\ F^\circ$

$f(84) = \frac{5}{9}(84-32)$

$= \frac{5}{9}.(52) = 28.89\ C^\circ$

$f^{-1}(x) = \frac{9}{5}x + 32$ (Convert from C° to F°)

$f^{-1}(28.89) = \frac{9}{5}\cdot(28.89) + 32 = 84\ F^\circ$

g ; g^{-1} ; u ; u^{-1}

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$\boxed{f(x) = \frac{5}{9}(x-32) \ ; \ f^{-1}(x) = \frac{9}{5}\boxed{x} + 32}$

$\boxed{f^{-1}(f(x))}$

$f(f^{-1}(x))$

$f^{-1}(\boxed{f(x)}) = f^{-1}\left(\boxed{\frac{5}{9}(x-32)}\right)$

$= \frac{9}{5}\cdot\left(\frac{5}{9}(x-32)\right) + 32$

$= x - 32 + 32 = x$

$f(f^{-1}(x)) = f\left(\frac{9}{5}x + 32\right)$

$= \frac{5}{9}\left(\frac{9}{5}x + \cancel{32} - \cancel{32}\right) = \underbrace{\frac{5}{9}\cdot\frac{9}{5}x}_{=x}$

$f^{-1}(f(x)) = x$; $f(f^{-1}(x)) = x$

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Definition of the inverse function.

Let $y = f(x)$ be a one-to-one function.

The inverse function of f is denoted by f^{-1} (read as f inverse)

It is the function which satisfies the following equations:

$$\boxed{\begin{aligned} f^{-1}(f(x)) &= x \\ f(f^{-1}(x)) &= x \end{aligned}}$$

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Procedure to test whether 2 functions are inverses of each other.

Suppose we have 2 functions $f(x)$ and $g(x)$.

(1) Find $f(g(x))$ and $g(f(x))$

(2) If $f(g(x)) = x$ and $g(f(x)) = x$, then f and g are inverses of each other.

Otherwise, they are not.

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E.g. $f(x) = \frac{1}{\boxed{x}+2}$; $g(x) = \frac{1}{x} - 2$

Is $g = f^{-1}$?

$f(g(x)) = f\left(\boxed{\frac{1}{x} - 2}\right)$

$= \frac{1}{\frac{1}{x} \cancel{-2} \cancel{+2}} = \frac{1}{\frac{1}{x}} = x$

$f(g(x)) = x$.

$g(f(x)) = g\left(\frac{1}{x+2}\right) = \frac{1}{\frac{1}{x+2}} - 2$

$= x + 2 - 2 = x$

$g(f(x)) = x$.

Yes, $g = f^{-1}$.

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