

4.1 Linear Functions

- ① What is a linear function? Determine whether a linear function is increasing, decreasing or constant.
- ② Find the slope of a linear function. Write an equation for a linear function.
- ③ Graph linear functions
- ④ Determine whether 2 lines are perpendicular or parallel. Write an equation of a line parallel or perpendicular to a given line.

Oct 6-10:00 AM

① What is a linear function?

A linear function is a function of the form

$$f(x) = mx + b$$

or

$$y = mx + b$$

E.g. $f(x) = 2x + 5$; $f(x) = -\frac{3}{7}x - \frac{12}{17}$

The graph of a linear function is a line.

Note: When $x = 0$, $y = b$. The point $(0, b)$ is called the y-intercept of f .

m is called the slope of $f(x)$.

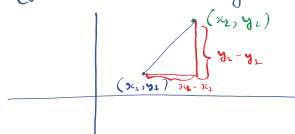
Find the R.O.C of $f(x)$ on $[x_1, x_2]$

Oct 6-10:05 AM

$$\text{R.O.C} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{mx_2 + b - (mx_1 + b)}{x_2 - x_1} = \frac{mx_2 + \cancel{b} - mx_1 - \cancel{b}}{x_2 - x_1} = \frac{m(x_2 - x_1)}{x_2 - x_1} = m$$

A.R.O.C. of f on $[x_1, x_2] = \frac{m x_2 - m x_1}{x_2 - x_1} = \frac{m(x_2 - x_1)}{x_2 - x_1} = m$

Constant rate of change = m .



Increasing / Decreasing / Constant

Oct 6-10:11 AM

$$f(x) = mx + b$$

If $m > 0$, then f is increasing.

If $m < 0$, then f is decreasing.

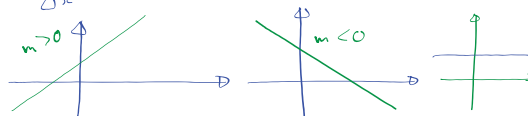
If $m = 0$, then f is constant.

$$f(x) = \frac{1}{2}x + 3$$

$$f(x) = -\frac{1}{3}x + \frac{5}{7}$$

$$f(x) = 10$$

$$m = \frac{\Delta y}{\Delta x} \quad \Delta y = m \Delta x$$



Note: We only need 2 points to determine a linear function.

Oct 6-10:16 AM

- ② Find the slope and an equation of a linear function given 2 points.

E.g. $f(x)$ is a linear function.

$(2, 3)$ and $(0, 4)$ are points on the line $y = f(x)$.

Find the slope of f . Find an equation for f .

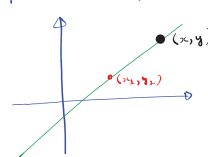
Determine whether f is increasing or decreasing.

Calculate the slope of a linear function given 2 points (x_1, y_1) ; (x_2, y_2) .

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Oct 6-10:21 AM

Point-Slope equation of a linear function



$$\frac{y - y_1}{x - x_1} = m$$

$$y - y_1 = m(x - x_1)$$

→ isolate y .

Solution to E.g.

$(2, 3)$ and $(0, 4)$.

$$\text{Slope} = \frac{4 - 3}{0 - 2} = \frac{1}{-2} = -\frac{1}{2}$$

$$\text{Point-Slope equation} \quad y - 3 = -\frac{1}{2}(x - 2)$$

Oct 6-10:26 AM