Linear function: $f(x) = mx + b$. m: slope, b: (0, b) y-intercept

$f(x) = 2x + 3$ $\quad f(x) = -x + 5$

$(x_1, y_1)$ ; $(x_2, y_2)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$

## 5.1 Quadratic Functions

Goal: Everything you need to know about quadratic functions

$$f(x) = ax^2 + bx + c, \quad a, b, c \text{ constants}, \ a \neq 0$$

E.g. $f(x) = 3x^2 + 5x - 7$

$f(x) = -\frac{1}{2}x^2 + \frac{3}{5}x - \frac{7}{12}$.

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* The graph of $f(x) = ax^2 + bx + c$ is a parabola

upward $a > 0$ $\quad$ downward $a < 0$

E.g. $f(x) = x^2 + 3x + 1$ ; $a = 1 > 0$.

$f(x) = -\frac{1}{3}x^2 + 5x - 4$ ; $a = -\frac{1}{3} < 0$.

Formula for the vertex of a parabola

vertex $(h, k)$ $\quad f(x) = ax^2 + bx + c$

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The x-coordinate of the vertex: $h = -\frac{b}{2a}$

The y-coordinate of the vertex: $k = f\left(-\frac{b}{2a}\right)$

$f(x) = ax^2 + bx + c$.

Vertex $(h, k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

E.g. $f(x) = 4x^2 + 2x - 7$

$h = \frac{-2}{2 \cdot 4} = \frac{-2}{8} = -\frac{1}{4}$

$k = f\left(-\frac{1}{4}\right) = 4 \cdot \left(-\frac{1}{4}\right)^2 + 2\left(-\frac{1}{4}\right) - 7$

$= 4 \cdot \frac{1}{16} + \left(-\frac{1}{2}\right) - 7$

$= \frac{1}{4} - \frac{1}{2} - 7 = \frac{1 - 2 - 28}{4} = -\frac{29}{4}$

Vertex $\left(-\frac{1}{4}, -\frac{29}{4}\right)$

$(h, k) \to$ vertex

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E.g. $f(x) = -5x^2 + 2x - 5$

$h = \frac{-2}{2(-5)} = \frac{-2}{-10} = \frac{1}{5}$.

$k = f\left(\frac{1}{5}\right) = -5\left(\frac{1}{5}\right)^2 + 2 \cdot \frac{1}{5} - 5$

$= -5 \cdot \frac{1}{25} + \frac{2}{5} - 5$

$= -\frac{1}{5} + \frac{2}{5} - 5 = \frac{-1 + 2 - 25}{5}$

$= -\frac{24}{5}$

Vertex $\left(\frac{1}{5}, -\frac{24}{5}\right)$

$(h, k)$

Why is the vertex given by such a formula?

(No need to know for exam)

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$f(x) = ax^2 + bx + c$

$= a\left(x^2 + \frac{b}{a}x\right) + c$

$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$

$= a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right) + c$

$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$

$a < 0$: $a\left(x + \frac{b}{2a}\right)^2 \leq 0$, $f(x) \leq k$

$a > 0$: $a\left(x + \frac{b}{2a}\right)^2 \geq 0$, $f(x) \geq k$

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Standard form of the parabola:

$f(x) = ax^2 + bx + c$.

Vertex $(h, k)$.

Standard form: $f(x) = a(x - h)^2 + k$

$f(x) = x^2 + x + 1$

Vertex $(h, k)$: $h = \frac{-1}{2(1)} = -\frac{1}{2}$.

$k = f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) + 1$

$= \frac{1}{4} - \frac{1}{2} + 1$

$= \frac{1 - 2 + 4}{4} = \frac{3}{4}$

Vertex $\left(-\frac{1}{2}, \frac{3}{4}\right)$

$f(x) = \left(x - \left(-\frac{1}{2}\right)\right)^2 + \frac{3}{4} = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4}$

$\left(x + \frac{1}{2}\right)^2 \geq 0$

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