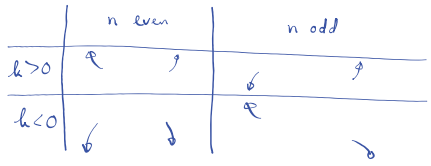
5.3 Graphs of Polynomial Functions

End behavior of polynomial functions

Key: End behavior of any polynomial function is the same as end behavior of its leading term.

$h \cdot x^n$



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Zeros and multiplicities of zeros

Definition of a zero of a polynomial function

A zero of a polynomial function $f(x)$ is a solution of the equation $f(x) = 0$. (A zero of f is just the x-value of an x-intercept)

E.g. $f(x) = (x+3)(x-2)^2(x+1)^3$

Find the zeros of f

$(x+3)(x-2)^2(x+1)^3 = 0$

$x+3=0$ or $(x-2)^2=0$ or $(x+1)^3=0$

$x=-3$ $x=2$ $x=-1$

$x=-3$ has multiplicity 1; $x=2$ has multiplicity 2; $x=-1$ has multi. 3

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Ex. Find the zeros and multiplicity of each zero of the given polynomial function.

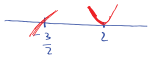
(a) $f(x) = (x-2)^2(2x+3)$

(b) $g(x) = x^3 - 5x^2 - x + 5$

(c) $h(x) = x^6 - 3x^4 + 2x^2$

Sol. (a) $f(x) = (x-2)^2(2x+3)$

$x = 2$ Mult. 2 $\quad x = -\frac{3}{2}$ Mult. 1



(b) $g(x) = x^3 - 5x^2 - x + 5 = 0$

$x^2(x-5) - (x-5) = 0$

$(x-5)(x^2-1) = 0$

$(x-5)(x-1)(x+1) = 0$

$x=5 \rightarrow$ mult. 1

$x=1 \rightarrow$ mult. 1

$x=-1 \rightarrow$ mult. 1

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(c) $h(x) = x^6 - 3x^4 + 2x^2$

$x^6 - 3x^4 + 2x^2 = 0$

$x^2(x^4 - 3x^2 + 2) = 0$

$x^2(x^2-1)(x^2-2) = 0$

$x^2(x-1)(x+1)(x-\sqrt{2})(x+\sqrt{2}) = 0$

$x=0$ Mult. 2 | $x=1$ Mult. 1 | $x=-1$ Mult. 1 | $x=\sqrt{2}$ Mult. 1 | $x=-\sqrt{2}$ Mult. 1.

Why care about the multiplicity of a zero?

Ans: The multiplicity of a zero tells you how the graph behaves at that zero.

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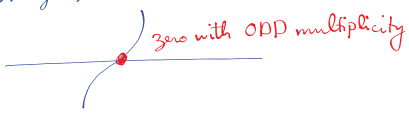
Graphical Behavior of a polynomial function at a zero (x-intercept)

(1) If a zero has EVEN multiplicity, when the graph reaches that zero, it will touch the zero and bounce back

(2) If a zero has ODD multiplicity, when the graph reaches that zero, it will cross that zero.



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Put everything to gether to graph a polynomial function.

Steps: (1) Find zeros (x-intercepts) and their multiplicities. Find y-intercept.

(2) Determine the end behavior. (Find and analyze the leading term)

(3) Use the multiplicities of zeros to determine the behavior at zero.

(4) Put everything together to sketch a rough graph of the function.

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