5.4. Dividing Polynomials

Goals: (1) Long Division for Polynomials
(2) Synthetic Division.

E.g. $(4x^3 + 12x^2 - x - 8) \div (2x+5)$ } divide

$$\underbrace{\frac{4x^3+12x^2-x-8}{2x+5}}$$

dividend, divisor

$$\frac{4x^3}{2x} = 2x^2 \qquad \frac{2x^2}{2x} = x \qquad \frac{-6x}{2x} = -3$$

$\boxed{\boxed{2x^2} + \boxed{x} - 3}$ → quotient

$\boxed{2x} + 5 \,\big|\, \boxed{4x^3} + 12x^2 - x - 8$

$-4x^3 - 10x^2$

$\boxed{\boxed{2x^2} - x - 8}$ → new dividend

$-2x^2 - 5x$

$\boxed{\boxed{-6x} - 8}$ → new dividend

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$-6x - 8$
$6x + 15$
$\boxed{7}$ → Remainder

$(4x^3 + 12x^2 - x - 8) \div (2x+5)$

Dividend Quotient $= 2x^2 + x - 3$

Remainder $= 7$

$$\frac{4x^3+12x^2-x-8}{2x+5} = \boxed{2x^2+x-3} + \frac{\boxed{7}}{2x+5}$$

Division, Quotient, Remainder

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Division algorithm.

Let $f(x)$ and $g(x)$ be polynomials.

Degree $g \le$ Degree f.

There exist unique polynomials $q(x)$ and $r(x)$ such that

$$\boxed{\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}}$$

$f(x) = q(x)g(x) + r(x)$

and degree of $r(x) <$ degree $g(x)$

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$(9x^3 + 7x^2 - x) \div (3x+7)$

Quotient: $\boxed{3x^2} + 4x$

Remainder: $\cancel{2x^2+27}$

$\boxed{3x^2 - \frac{14}{3}x + \frac{95}{9}}$ ← Q

$\boxed{3x} + 7 \,\big|\, \boxed{9x^3} + 7x^2 - x$

$-9x^3 - 21x^2$

$\boxed{-14x^2} - x$

$14x^2 + \frac{98}{3}x$

$$\frac{9x^3}{3x} = 3x^2 \qquad \frac{-14x^2}{3x} = \qquad \left(-\frac{14}{3}x\right)$$

Q: $\boxed{3x^2} - \frac{14}{3}x - 33$

R: 0

$-\frac{14}{3}x \cdot 7$

$\frac{95}{3}x$

$-\frac{95}{3}x + \frac{665}{9}$

$$\frac{\frac{95}{3}x}{3x} \qquad \frac{\frac{95}{3}}{3} = \frac{95}{3}\cdot\frac{1}{3} = \frac{95}{9}$$

$-\frac{98}{3}x$

$95 \times 7 = 665$

$-x + \frac{98}{3}x$

$\left(-1 + \frac{98}{3}\right)x$

$\frac{95}{3}$

$\boxed{\frac{665}{9}}$ → R.

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Synthetic Division

If we divide $f(x)$ by $x - a$ or $x + a$ $(a > 0)$, then we can use a short cut called synthetic division instead of long division.

* Situation: divide by $x - a$.

$(\boxed{-9}x^4 + \boxed{10}x^3 + \boxed{7}x^2 - 6) \div (x - \boxed{1})$

1	−9	10	7	0	−6
		−9	1	8	8
	−9	1	8	8	2

Remainder

$\boxed{-9x^3 + x^2 + 8x + 8}$

Quotient

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$$\boxed{\frac{-9x^4 + 10x^3 + 7x^2 - 6}{x - 1} = -9x^3 + x^2 + 8x + 8 + \frac{2}{x-1}}$$

Situation: $x + a$

E.g. $(3x^4 + 18x^3 - 3x + 40) \div (x+2)$

$x - (-2)$

−2	3	18	0	−3	40
		−6	−24	48	−90
	3	12	−24	45	−50

R

Quotient

$\boxed{-6x^3 - 24x^2 + 48x + 90}$

R: $\boxed{130}$

$3x^3 + 12x^2 - 24x + 45$

R: $\boxed{-50}$

Quotient: $3x^3 + 12x^2 - 24x + 45$

Remainder: -50

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