(3) To find the zeros
$x^3 - 6x^2 - x + 30 = 0$
$(x+2)(x^2 - 8x + 15) = 0$
Either $x+2=0$ or $x^2 - 8x + 15 = 0$
$\boxed{x=-2}$ $(x-3)(x-5) = 0$
$\boxed{x=3}$; $\boxed{x=5}$

Ex: $f(x) = x^3 + 4x^2 - 4x - 16$
(1) Verify that $x-2$ is a factor of $f(x)$
(2) Find the other factor
(3) Find all zeros of $f(x)$

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(1) $(x^3 + 4x^2 - 4x - 16) \div (x-2)$

$$\begin{array}{c|cccc} 2 & 1 & 4 & -4 & -16 \\ & & 2 & 12 & 16 \\ \hline & 1 & 6 & 8 & \boxed{0} \end{array}$$
← Remainder.

(2) Quotient: $x^2 + 6x + 8$
$(x^3 + 4x^2 - 4x - 16) = (x^2 + 6x + 8)(x-2)$
(3) $x^3 + 4x^2 - 4x - 16 = 0$
$(x-2)(x^2+6x+8) = 0$
$x-2=0$ or $x^2 + 6x + 8 = 0$
$\boxed{x=2}$ $(x+4)(x+2) = 0$
$\boxed{x=-4}$; $\boxed{x=-2}$

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Rational Zero Theorem (list all the possibilities for rational zeros of a polynomial function)

The Rational zero Theorem

If a polynomial $f(x) = \boxed{a_n} x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + \boxed{a_0}$ has integer coefficients, then every rational zero of $f(x)$ has the form $\frac{p}{q}$ where p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n

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E.g. list all the possibilities for rational zeros of $f(x) = 2x^4 - 5x^3 + x^2 - 4$.
Any rational zero of f has the form $\frac{\pm 4}{\pm 2}$; $\frac{\pm 4}{\pm 1}$

$\frac{p}{q} = \frac{\pm 1 ; \pm 2 ; \pm 4}{\pm 2 ; \pm 1}$; $\boxed{\frac{\pm 1}{\pm 1}}$, $\frac{\pm 1}{\pm 2}$; $\frac{\pm 2}{\pm 2}$, $\frac{\pm 2}{\pm 1}$
$\boxed{1, -1 ; \frac{1}{2}, -\frac{1}{2} ; 2, -2 ; 4, -4}$

Factors of -4: $\pm 1, \pm 2 ; \pm 4$
Factors of 2. $\pm 2 ; \pm 1$

E.g. Use the Rational Zero Theorem to find rational zeros of $f(x) = 2x^3 + x^2 - 4x + 1$.

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list the possibilities of rational zeros.
$\frac{p}{q} = \frac{\pm 1}{\pm 1 ; \pm 2}$; $\frac{\pm 1}{\pm 1}$; $\frac{\pm 1}{\pm 2}$
$\boxed{\boxed{1}, -1 ; \frac{1}{2}, -\frac{1}{2}}$
p : factor of 1 Factors 1 : ± 1
q : factor of 2. Factors of 2 : $\pm 1, \pm 2$
$\boxed{f(x) = 2x^3 + x^2 - 4x + 1}$
$f(1) = 2 + 1 - 4 + 1 = 0 \longrightarrow 1$ is the only rational zero of $f(x)$
$f(-1) = -2 + 1 + 4 + 1 = 4$
$f(\frac{1}{2}) = 2 \cdot \frac{1}{8} + \frac{1}{4} - 2 + 1 = -\frac{1}{2}$
$f(-\frac{1}{2}) = 2(-\frac{1}{8}) + \frac{1}{4} + 2 + 1 = 3$

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Since 1 is a zero, by the Factor Theorem,
$x-1$ is a factor of f.
$(2x^3 + x^2 - 4x + 1) \div (x-1)$

$$\begin{array}{c|cccc} 1 & 2 & 1 & -4 & 1 \\ & & 2 & 3 & -1 \\ \hline & 2 & 3 & -1 & \boxed{0} \end{array}$$

Quotient: $2x^2 + 3x - 1$
$(2x^3 + x^2 - 4x + 1) = (x-1)(2x^2 + 3x - 1) = 0$
$2x^2 + 3x - 1 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-3 \pm \sqrt{9 - 4 \cdot 2 \cdot (-1)}}{4} = \boxed{\frac{-3 \pm \sqrt{17}}{4}}$

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Fundamental Theorem of Algebra

If $f(x)$ is a polynomial of degree n, then f has exactly n complex zeros (counting multiplicities)

In other words, f can be factored into linear factors

$$f(x) = a(x - c_1)(x - c_2)\cdots(x - c_n)$$

$c_1, c_2, \ldots, c_n$: complex zeros.

Complex conjugate theorem

If $a + bi$ is a zero of $f(x)$, then $a - bi$ is also a zero of f.

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E.g. $2 + i$ is a zero of f
$2 - i$ will also be a zero

$3 + 5i$ is zero ; $3 - 5i$ will also be a zero

$7i$ is zero, $-7i$ ____________

E.g. Find a poly. given that it has zeros
$-3, 2, \boxed{i}$; $-i$

$(x+3)(x-2)(x-i)(x+i)$

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