5.5 Zeros of Polynomials

Useful result to find zeros of polynomials

Goals: (1) Remainder Theorem
(2) Factor Theorem
(3) Rational zeros Theorem
(4) Fundamental Theorem of algebra
(5) Complex Conjugate Theorem

What is a zero of a polynomial?

We say that a number k is a zero (or root) of a polynomial $f(x)$ if $f(k) = 0$

Nov 1-10:00 AM

1) Remainder Theorem

E.g. $(\underbrace{6x^4 - x^3 - 15x^2 + 2x - 7}_{f(x)}) \div (x - 1)$

What is the remainder?

$$\begin{array}{r|rrrrr} 1 & 6 & -1 & -15 & 2 & -7 \\ & & 6 & 5 & -10 & -8 \\ \hline & 6 & 5 & -10 & -8 & \boxed{-15} \end{array} \longrightarrow \text{Remainder.}$$

Remainder Theorem says that to find the remainder, we can also evaluate f at 1. $f(1) = 6 - 1 - 15 + 2 - 7 = \boxed{-15}$

Nov 1-10:08 AM

Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is equal to $\boxed{f(k)}$

Why is this true?

$f(x) \div (x - k) \longrightarrow$ quotient $q(x)$; remainder r

$$\underbrace{f(x)}_{\text{Dividend}} = \underbrace{(x - k)}_{\text{divisor}}\underbrace{q(x)}_{\text{quotient}} + \underbrace{r}_{\text{remainder.}}$$

$$f(k) = \underbrace{(k - k)\,q(k)}_{0} + r$$

$$\boxed{f(k) = r}$$

Nov 1-10:14 AM

* Use the remainder Theorem to evaluate polynomial.

Eg. $f(x) = 2x^5 - 3x^4 - 9x^3 + 8x^2 + 2$

Evaluate f at $x = -3$. (Find $f(-3)$)

Remainder Theorem says that

$\boxed{f(-3)}$ = remainder when we divide $(2x^5 - 3x^4 - 9x^3 + 8x^2 + 2)$ by $(x - (-3))$

$(2x^5 - 3x^4 - 9x^3 + 8x^2 + 2) \div (x + 3)$

$$\begin{array}{r|rrrrrr} -3 & 2 & -3 & -9 & 8 & 0 & 2 \\ & & -6 & 27 & -54 & 138 & -414 \\ \hline & 2 & -9 & 18 & -46 & 138 & \boxed{-412} \end{array} = f(-3)$$

Nov 1-10:20 AM

Factor Theorem

k is a zero of $f(x)$ if and only if $x - k$ is a factor of $f(x)$

Why is this true?

$f(x) \div (x - k)$

Remainder Theorem says Remainder $= f(k)$

k is a zero of $f(x)$ iff $f(k) = 0$ iff Remainder when $f(x)$ is divided by $x - k$ $= 0$ iff $x - k$ is a factor of $f(x)$

Nov 1-10:28 AM

E.g. Use Factor Theorem to find zeros of polynomials.

(1) Verify that $x + 2$ is a factor of $x^3 - 6x^2 - x + 30$

(2) Find the other factor

(3) Find all the zeros of the polynomial.

(1) $(x^3 - 6x^2 - x + 30) \div (x + 2)$

$$\begin{array}{r|rrrr} -2 & 1 & -6 & -1 & 30 \\ & & -2 & 16 & -30 \\ \hline & 1 & -8 & 15 & \boxed{0} \end{array} \longrightarrow \text{Remainder} = 0$$

Since the remainder $= 0$, $x + 2$ is a factor of $x^3 - 6x^2 - x + 30$

(2) Quotient: $x^2 - 8x + 15$

$x^3 - 6x^2 - x + 30 = (x + 2)\cdot(x^2 - 8x + 15)$

Nov 1-10:32 AM