

6.1. Exponential Function

E.g. \$1000. Invest in an account which pays 6% return a year. How much money will you get after 30 years?

\$1000

Year 1: $1000 \cdot (1.06)$

Year 2: $1000 \cdot (1.06) \cdot (1.06) = 1000 \cdot (1.06)^2$

Year 3: $1000 \cdot (1.06)^3$

Year x: $1000 \cdot (1.06)^x$

Year 30: $1000 \cdot (1.06)^{30} = 5743.49$

base (pointing to 1.06)
exponent (pointing to x)

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Definition of an exponential function:

An exponential function is a function of the form

$$f(x) = a \cdot (b)^x$$

a: non-zero constant, a is called the initial value.

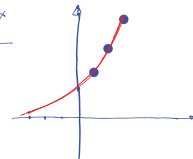
b: positive #, $b \neq 1$, b is called the base.

E.g. $f(x) = 1000 \cdot (1.06)^x$

Evaluate exponential function by hand

$$f(x) = 2^x$$

x	$f(x) = 2^x$
-3	$2^{-3} = \frac{1}{8}$
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	2
2	4
3	8



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* Exponential growth model

$$f(x) = ab^x$$

E.g. Population growth.

Population of India is 1.25 billion people in 2013 with an annual growth rate of 1.2% a year.

Q1. Find an exponential function that represents this population over time.

Q2. Find the population of India in 2034.

1.25

Year 1: $(1.25) \cdot (1.012)$ Year x: $(1.25)(1.012)^x$

Year 2: $(1.25) \cdot (1.012)^2$

$f(x) = 1.25 \cdot (1.012)^x$

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Year 2031 • $x=18$; $f(18) = (1.25)(1.012)^{18} = 1.549$ billion.

Ex: Population of China in 2013 is 1.39 billion.

Annual growth rate of this population 0.6%

Q1. Find exp. function which represents this situation.

Q2: Find the population of China in 2040.

$$f(x) = 1.39 \cdot (1.006)^x$$

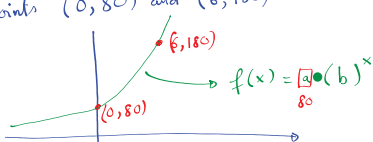
$$f(27) = (1.39) \cdot (1.006)^{27}$$

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Find equations of exponential function given 2 data points

$$f(x) = a \cdot b^x$$

E.g. Find an exponential function that passes through 2 points (0, 80) and (6, 180)



Since (0, 80) belongs to the graph, when $x=0$, $f(0)=80$

$$f(0) = a \cdot b^0 = 80$$

$$a = 80$$

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Since (6, 180) belongs to the graph,

$$f(6) = 180 \quad f(x) = a \cdot b^x$$

$$f(6) = a \cdot b^6 = 180$$

$$80 \cdot b^6 = 180$$

$$b^6 = \frac{180}{80} = \frac{9}{4}$$

$$b^6 = \frac{9}{4}$$

$$b = \left(\frac{9}{4}\right)^{\frac{1}{6}} \approx 1.1447$$

$$a = 80; b = 1.1447$$

$$f(x) = 80 \cdot (1.1447)^x$$

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