6.4 Graphs of Logarithmic Functions

$\log_{\boxed{b}}(x) = y$ is equivalent $b^y = x$

Domains of Logarithmic Functions

E.g. $f(x) = \log_2(x)$

$\log_2(\boxed{x}) = y \iff 2^y = x$

x must be positive

So, the domain of $f(x) = \log_2(x)$ is the set of positive numbers. In interval notation, $(0, \infty)$

$\boxed{\text{In general, Domain of } f(x) = \log_b(x) \text{ is } (0, \infty)}$

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E.g. $\log_2(2x-3)$ or $\log_2(5-3x)$, $\ln(-x+7)$

Key: $\log_b(\boxed{\text{Stuff}})$

To find the domain, we solve the inequality

$\boxed{\text{Stuff} > 0}$

E.g. Find the domain of $\log_2(\boxed{2x-3})$

Want: $2x-3 > 0$, $x > \frac{3}{2}$.

Domain: $\left(\frac{3}{2}, \infty\right)$

E.g. Find the domain of $\log(\boxed{(2x-3)(x-1)})$

Want: $(2x-3)(x-1) > 0$

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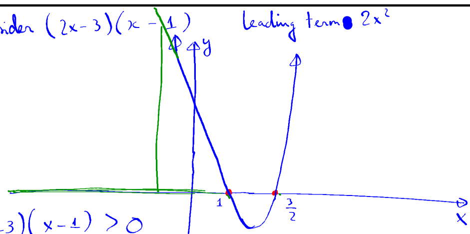
Consider $(2x-3)(x-1)$ leading term: $2x^2$



$(2x-3)(x-1) > 0$

when x is in

$\boxed{(-\infty, 1) \cup \left(\frac{3}{2}, \infty\right)}$

Ex: Find the domain of the given function

(a) $f(x) = \ln(5-3x)$ (b) $g(x) = \log_{\frac{1}{2}}(x^2-x-6)$

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Sol: (1) $f(x) = \ln(5-3x)$

$5 - 3x > 0$

$-3x > -5$

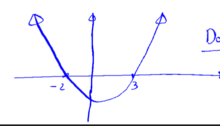
$x < \frac{-5}{-3} = \frac{5}{3}$

Domain: $\left(-\infty, \frac{5}{3}\right)$

(2) $f(x) = \log_{\frac{1}{2}}(x^2-x-6)$

$x^2 - x - 6 > 0$

$(x-3)(x+2) > 0$



Conclusion:

Domain: $(-\infty, -2) \cup (3, \infty)$

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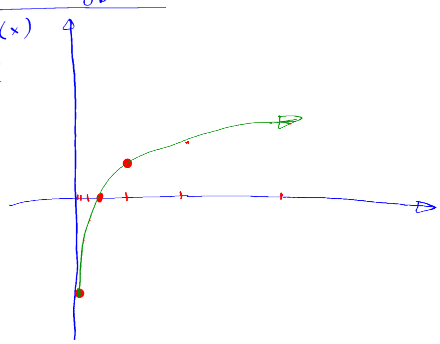
Graphs of $f(x) = \log_b(x)$

$f(x) = \log_2(x)$

x	$f(x)$
8	3
4	2
2	1
1	0
$\frac{1}{2}$	-1
$\frac{1}{4}$	-2
$\frac{1}{8}$	-3



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In general, the graph of $f(x) = \log_b(x)$, $b > 1$ looks like

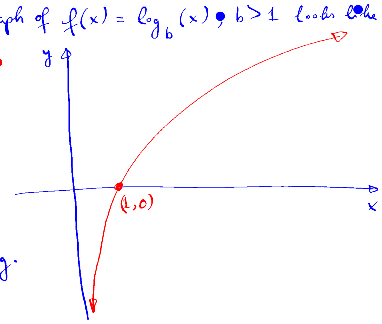
Vertical asymptote: $x = 0$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

x-intercept $(1, 0)$

function is increasing.



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