6.5. Logarithmic Properties

(1) Inverse Property

$$\boxed{\log_b\left(b^x\right) = x \qquad b^{\log_b(x)} = x}$$

E.g. $\log\left(10^{2016}\right) = 2016$

$\ln(e^3) = 3$

$\ln(e^x) = x$

$\ln(e^{\text{Stuff}}) = \text{Stuff}$

$10^{\log(5)} = 5$

$e^{\ln(2x+3)} = 2x+3$

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(2) Product Rule

Product Rule says that log of a product is equal to the sum of the individual logs.

$$\boxed{\log_b(M \cdot N) = \log_b(M) + \log_b(N)}$$

E.g. $\log(\boxed{100}\cdot\boxed{x}) = \log 100 + \log x = 2 + \log x$

$\log(100x) = 2 + \log x$

E.g. $\log_3(\boxed{30x}\cdot\boxed{(3x+4)}) \longrightarrow$ expand as much as possible.

$= \log_3(30x) + \log_3(3x+4)$

$= \log_3(30) + \log_3(x) + \log_3(3x+4)$

$= \log_3(3) + \log_3(10) + \log_3(x) + \log_3(3x+4)$

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$= 1 + \log_3(2) + \log_3(5) + \log_3(x) + \log_3(3x+4)$

Power Rule

$$\boxed{\log_b\left(M^{\boxed{P}}\right) = p \cdot \log_b(M)}$$

E.g. $\log_2\left(x^5\right) = 5 \cdot \log_2(x)$.

Why is this true? $\log_b(M^2) = \log_b(M \cdot M)$

$= \log_b(M) + \log_b(M)$

$= 2\log_b(M)$

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Quotient Rule

Quotient Rule says that log of a quotient will be equal to the difference of individual logs.

$$\boxed{\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)}$$

E.g. $\log_3\left(\frac{\boxed{7x^2+21x}}{\boxed{7x\cdot(x-1)}}\right) \longrightarrow$ expand as much as possible.

$= \log_3(\boxed{7x^2+21x}) - \log_3(\boxed{7x}\cdot\boxed{(x-1)})$

$= \log_3(\boxed{7x}\boxed{(x+3)}) - \log_3(\boxed{7x}\boxed{x-1})$

$= \log_3(7x) + \log_3(x+3) - \left(\log_3(7x) + \log_3(x-1)\right)$

$= \cancel{\log_3(7x)} + \log_3(x+3) - \cancel{\log_3(7x)} - \log_3(x-1)$

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$= \log_3(x+3) - \log_3(x-1)$

$\log_b(b^x) = x$, $b^{\log_b(x)} = x$

$\log_b(MN) = \log_b(M) + \log_b(N)$

$\log_b(M^P) = p\log_b(M)$

$\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$

Expand a log expression.

Condense a log expression

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Expanding log Expressions

E.g. $\log_6\left(\frac{6x^3}{y^2 3}\right) \longrightarrow$ expand it as much as possible.

$= \log_6(6) + \log_6(x^3) - \log_6(y^2) - \log_6(3)$

$= 1 + 3\log_6(x) - 2\log_6(y) - \log_6(3)$.

E.g. $\log\left(\sqrt{x}\right) = \log\left(x^{\frac{1}{2}}\right) = \frac{1}{2}\log(x)$.

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