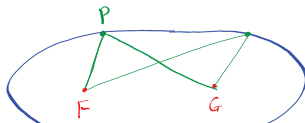


10.1 Ellipses.

Conic Sections

- Ellipses
- Hyperbolas
- Parabolas

Definition of an ellipse:

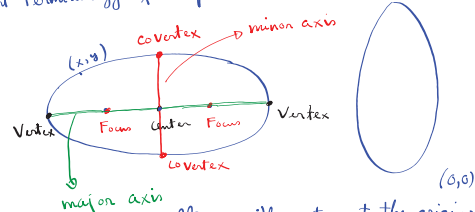


Fix 2 points F and G in the plane, called foci. The ellipse with foci at F and G is the collection of points in the plane P such that $PF + PG = \text{constant}$.

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such that $PF + PG = \text{constant}$

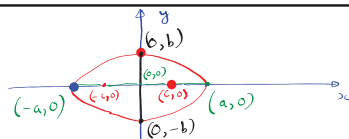
Important terminology of ellipses



Standard equation of an ellipse with center at the origin.

Case 1: Major axis is on the x -axis

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Standard equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $a > b > 0$

Coordinates of vertices: $(a, 0)$ and $(-a, 0)$

length of major axis is $2a$

Coordinates of covertices: $(0, b)$ and $(0, -b)$

length of minor axis is $2b$

Coordinates of focus points: $(c, 0)$ and $(-c, 0)$
where $c^2 = a^2 - b^2$

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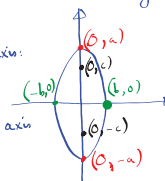
② Major axis is on the y -axis

length of major axis:

$2a$

length of minor axis:

$2b$



Standard equation:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

where $a > b > 0$.

Vertices: $(0, a)$ and $(0, -a)$

Covertices: $(b, 0)$ and $(-b, 0)$

Foci: $(0, c)$ and $(0, -c)$

$$c^2 = a^2 - b^2$$

E.g. $\frac{x^2}{49} + \frac{y^2}{25} = 1$ (Major axis on x -axis)

Q: Find vertices, covertices, focus points of this ellipse

Vertices: $(7, 0)$ and $(-7, 0)$

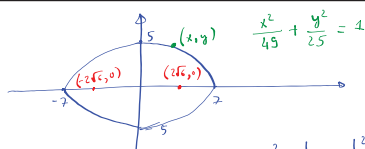
Covertices: $(0, 5)$ and $(0, -5)$

Foci: $(2\sqrt{6}, 0)$ and $(-2\sqrt{6}, 0)$

$$c^2 = 49 - 25 = 24$$

$$c = \sqrt{24} = 2\sqrt{6}$$

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E.g.

$$9x^2 + 16y^2 = 1$$

$$\frac{x^2}{\frac{1}{9}} + \frac{y^2}{\frac{1}{16}} = 1$$

$$c^2 = \frac{1}{9} - \frac{1}{16} = \frac{16-9}{144}$$

$$c = \frac{\sqrt{7}}{12}$$

$$a^2 = \frac{1}{9}; b^2 = \frac{1}{16}$$

$$a = \frac{1}{3}; b = \frac{1}{4}$$

(major axis on x -axis)

Vertices: $(\frac{1}{3}, 0)$ and $(-\frac{1}{3}, 0)$

Covertices: $(0, \frac{1}{4})$ and $(0, -\frac{1}{4})$

Foci: $(\frac{\sqrt{7}}{12}, 0)$ and $(-\frac{\sqrt{7}}{12}, 0)$

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Ex: $9x^2 + 4y^2 = 36$ (Ellipse on not)

If yes, Find vertices, covertices and focus points.

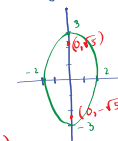
$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \quad (\text{major axis on } y\text{-axis})$$

Vertices: $(0, 3)$ and $(0, -3)$

Covertices: $(-2, 0)$ and $(2, 0)$

$$c^2 = 5; c = \pm\sqrt{5}$$

Foci: $(0, -\sqrt{5})$ and $(0, \sqrt{5})$



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