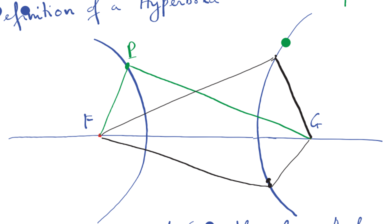
10.2 Hyperbolas

$|PF - PG| = \text{constant}$

Definition of a Hyperbola



Fix 2 points F and G in the plane. A hyperbola with focus points at F and G is the collection of points P in the plane such that $|PF - PG| = \text{constant}$.

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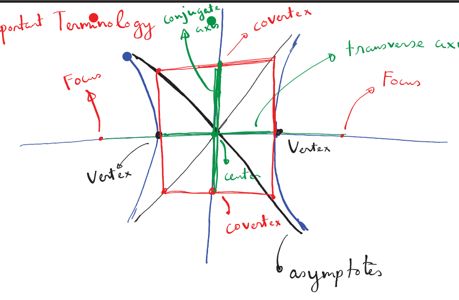
Important Terminology



* Standard form of the equation of a hyperbola with center (0,0).

(1) Transverse axis is on x-axis

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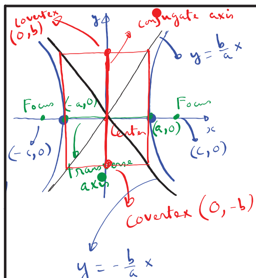

Equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Vertices: $(a,0)$, $(-a,0)$

length of transverse axis: $2a$

length of the conjugate axis: $2b$

Covertices: $(0,b)$, $(0,-b)$

Focus points: $(c,0)$, $(-c,0)$

where $c^2 = a^2 + b^2$

Distance between 2 focus points: $2c$.

Equations of asymptotes

$y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$

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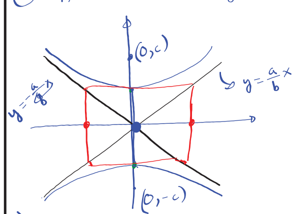
(2) Transverse axis on y-axis



Equation: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Vertices: $(0,a)$, $(0,-a)$

Covertices: $(b,0)$; $(-b,0)$

Focus points: $(0,c)$; $(0,-c)$

$c^2 = a^2 + b^2$.

Equations of asymptotes

$y = \frac{a}{b}x$; $y = -\frac{a}{b}x$

Ex: (1) Find the vertices and foci of the hyperbola

(a) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

(b) $\frac{y^2}{49} - \frac{x^2}{32} = 1$

(2) Find the standard form of the hyperbola that has vertices $(\pm 6, 0)$ and foci $(\pm 2\sqrt{10}, 0)$.

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(a) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

Vertices: $(3,0)$, $(-3,0)$

$a^2 = 9$; $b^2 = 25$; $c^2 = a^2 + b^2 = 9 + 25 = 34$

Focus: $(\sqrt{34}, 0)$; $(-\sqrt{34}, 0)$

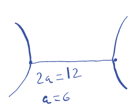
(b) $\frac{y^2}{49} - \frac{x^2}{32} = 1$.

Vertices: $(0,7)$, $(0,-7)$

$c^2 = a^2 + b^2 = 49 + 32 = 81$

Focus: $(0,9)$; $(0,-9)$

$\boxed{\frac{x^2}{36} - \frac{y^2}{4} = 1}$



(2) Given: Vertices $(\pm 6, 0)$; Focus $(\pm 2\sqrt{10}, 0)$

Find equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $a = 6$; $c = 2\sqrt{10}$

$b^2 = c^2 - a^2 = 40 - 36 = 4$

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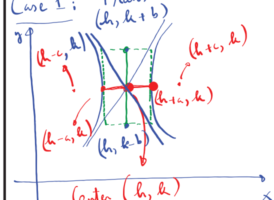
Hyperbolas with center at (h,k)

Case 1: Transverse axis is // x-axis



Standard equation:

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Vertices: $(h-a, k)$; $(h+a, k)$

Covertices: $(h, k+b)$; $(h, k-b)$

Focus: $(h+c, k)$, $(h-c, k)$

Asymptotes: $\boxed{y = \pm\frac{b}{a}(x-h) + k}$

Case 2: Transverse axis is // y-axis.

Standard equation: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

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