11.1. Sequences

Definition: A sequence is a list of numbers.

E.g. $\{2, 4, 8, 16, 32, 64, 128, \dots\}_{\infty}$ — infinite sequence

$a_1 \; a_2 \; a_3 \; a_4 \; a_5 \; a_6 \; a_7 \quad a_{100} = 2^{100}$

The elements of the list are called the terms of the sequence

First term of a sequence is denoted by a_1 (or b_1; c_1; x_1)

$a_1 = 2$

Second ______ a_2 (or b_2; c_2; x_2)

$a_2 = 4$

n^{th} term of a sequence is denoted by $\boxed{a_n}$ → (also called the general term of the sequence), $a_n = 2^n$

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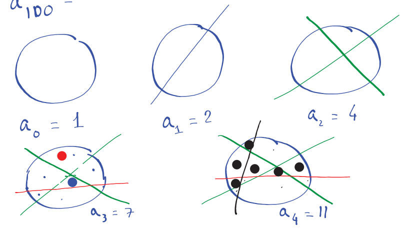
Many sequences are given by specifying an explicit formula for the general term a_n.

E.g. $\boxed{a_n = -3n + 8}$

$a_1 = 5$; $a_2 = 2$; $a_{10} = -22$

$a_{100} = -3 \cdot 100 + 8 = -292$

E.g.



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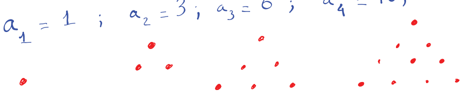
a_n = maximum # of pieces formed when slicing a pancake with n cuts

E.g. Sequence of triangular numbers.

$$a_n = \frac{n(n+1)}{2}$$

$a_1 = 1$; $a_2 = 3$; $a_3 = 6$; $a_4 = 10$; ...



Ex. (1) Consider the sequence defined as

$$a_n = \frac{\boxed{(-1)^n} \cdot n^2}{n+1}$$

Find $a_1 = ?\; -\frac{1}{2}$ $a_2 = ?\; \frac{4}{3}$ $a_3 = ?\; -\frac{9}{4}$ $a_4 = ?\; \frac{16}{5}$, $a_5 = ?\; -\frac{25}{6}$

What do you notice? Alternating sequence

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(2) $a_n = \frac{4n}{(-2)^n}$

$a_1 = ?\; -2$ $a_2 = ?\; 2$ $a_3 = ?\; -\frac{3}{2}$ $a_4 = ?\; 1$ $a_5 = ?\; -\frac{5}{8}$

Alternating Sequence.

Sequences where a_n is given by a piecewise explicit formula.

$$a_n = \begin{cases} n^2 & \text{if } 3 \nmid n \\ \frac{n}{3} & \text{if } 3 \mid n \end{cases}$$

$a_1 = 1$; $a_2 = 4$; $a_3 = 1$; $a_4 = 16$; $a_5 = 25$

$a_6 = 2$.

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$$a_n = \begin{cases} 2n^3 & \text{if } n \text{ is odd} \\ \left[\frac{5n}{2}\right] & \text{if } n \text{ is even} \end{cases}$$

$a_{20} = 50$; $a_3 = 54$

In many situations, an explicit formula for a_n is not given, we need to find it.

$\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \dots\}$

$\boxed{a_n = \frac{1}{n^2}}$ (Cal 2: $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots = \frac{\pi^2}{6}$)

$a_n = \frac{1}{n^n}$; $a_1 = \frac{1}{1}$ ✓; $a_2 = \frac{1}{2^2} = \frac{1}{4}$; $a_3 = \frac{1}{27}$

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$\{-1, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}, -\frac{1}{25}, \frac{1}{36}, \dots\}$

$$a_n = (-1)^n \frac{1}{n^2} = \frac{(-1)^n}{n^2}$$

$\{1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, -\frac{1}{36}, \dots\}$

$$a_n = -\frac{(-1)^n}{n^2} = \frac{(-1)^1 \cdot (-1)^n}{n^2} = \boxed{\frac{(-1)^{n+1}}{n^2}}$$

Ex: Find a formula for a_n for the given sequence. $\boxed{a_n = \frac{-2}{5^{n+1}}}$

(a) $\{-\frac{2}{25}, -\frac{2}{125}, -\frac{2}{625}, -\frac{2}{3125}, \dots\}$

(b) $\{-\frac{2}{25}, \frac{2}{125}, -\frac{2}{625}, \frac{2}{3125}, \dots\}$

$(-1)^{n+1} \cdot \frac{(-2)}{5^{n+1}}$ $\quad (-1)^n \cdot \frac{2}{5^{n+1}}$

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