(c) $\left\{-\frac{3}{4}, \frac{9}{8}, -\frac{27}{12}, \frac{81}{16}, -\frac{243}{20}, \ldots\right\}$

$a_n = (-1)^n \frac{3^n}{4n} = \frac{(-3)^n}{4n}$

$(-1)^n \cdot 3^n = \left((-1)\cdot 3\right)^n = (-3)^n$

In many situations, a sequence is given by a recursive formula. It means that a term is defined by previous terms.

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E.g. of a recursively defined sequence

Fibonacci Sequence

$\{1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots\}$

$a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$

$a_1 = 1$, $a_2 = 1$

Population of rabbits

1 newly born pair of rabbits

1 adult pair of rabbits

2 pairs

3 pairs

5 pairs

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Ex: (1) Given a recursively defined sequence

$a_1 = 1$, $a_2 = 2$.

$a_n = 3a_{n-1} + 4a_{n-2}$ for $n \geq 3$.

Find the first 6 terms of this sequence.

$a_3 = 3a_2 + 4a_1 = 10$

$a_4 = 3a_3 + 4a_2 = 30 + 8 = 38$

$a_5 = 3a_4 + 4a_3 = 3 \cdot 38 + 4 \cdot 10$

(2) $a_1 = 0$; $a_2 = 1$; $a_3 = 1$

$a_n = \frac{a_{n-1}}{a_{n-2}} + a_{n-3}$ for $n \geq 4$

Find the first 6 terms of the sequence.

$a_4 = \frac{a_3}{a_2} + a_1 = \frac{1}{1} + 0 = 1$

$a_5 = \frac{a_4}{a_3} + a_2 = \frac{1}{1} + 1 = 2$.

$a_6 = \frac{a_5}{a_4} + a_3 = \frac{2}{1} + 1 = 3$

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$a_0 = 1$, $a_1 = 1$

$1! = 1 \cdot 0! = 1$

$a_n = n \cdot a_{n-1}$ for $n \geq 2$

$a_{10} = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \ldots$

$a_2 = 2 \cdot a_1 = 2 \cdot 1 = 2$

$a_3 = 3 \cdot a_2 = 3 \cdot 2 \cdot 1 = 3! = 6$

$a_4 = 4a_3 = 4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24$

$a_5 = 5a_4 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120$

$2! = 2 \cdot 1 = 2$

$3! = 3 \cdot 2 \cdot 1 = 6$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$a_n = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdots 1 = n!$

(n factorial)

2 letters $\{a, b\}$: ab ; ba: $2 = 2!$

3 letters $\{a, b, c\}$: abc, acb, bac, bca, cab, cba: $6 = 3!$

4 letters $\{a, b, c, d\}$: $4! = 24$

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$n!$ = # of ways to arrange n things

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