## 11.4. Summation Notations and Series

* Summation Notation

$\{2, 4, 8, 16, 32, 64, \dots\} \leftarrow$ infinite Sequence.

General term $a_n = \boxed{2^n.}$

$2 + 4 + 8 + 16 + 32 + 64 + \dots + 2^{20}$

$= a_1 + a_2 + a_3 + \dots + a_{20}$

Summation Notation (Sigma Notation)

$$\sum_{n=1}^{20} \boxed{a_n}$$

20 → upper limit of summation

$a_n$ → summand

1 → lower limit of summation

n → index of summation

Nov 9-6:30 PM

$$\sum_{n=1}^{20} a_n = a_1 + a_2 + a_3 + \dots + a_{20}.$$

$$\sum_{n=1}^{20} 2^n = 2^1 + 2^2 + 2^3 + 2^4 + \dots + 2^{20}$$

Ex. (1) Evaluate: (a) $\sum_{k=1}^{7} k^2$ (b) $\sum_{j=2}^{6} (3j - 1)$

(2) Write the sum in summation notation

(a) $5 + 10 + 15 + 20 + \dots + 50$

(b) $\frac{3}{2} + \frac{3}{4} + \frac{3}{6} + \frac{3}{8} + \frac{3}{10} + \dots \frac{3}{100}$

(1) (a) 140 ; (b) 55

Nov 9-6:36 PM

(2) (a) $5 + 10 + 15 + 20 + \dots + 50$

$$\boxed{\sum_{x=1}^{10} 5x}$$

(b) $\frac{3}{2} + \frac{3}{4} + \frac{3}{6} + \dots + \frac{3}{100}$

$$\boxed{\sum_{x=1}^{50} \frac{3}{2x}}$$

Series is "the" sum of all the terms of an infinite sequence

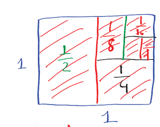
Nov 9-6:44 PM

$\{2, 4, 8, 16, 32, \dots\}$ $a_n = 2^n$

$$2 + 4 + 8 + 16 + \dots = \sum_{n=1}^{\infty} 2^n = \infty$$

$\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots\}$ $a_n = \frac{1}{2^n}$

$$\boxed{\sum_{n=1}^{\infty}} \frac{1}{2^n} = 1$$




Partial Sum of a series

Nov 9-6:47 PM

$\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots\}$

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$

$S_1 = \frac{1}{2}$ (first partial sum of the series)

$S_2 = \frac{1}{2} + \frac{1}{4}$ (second partial sum of the series = sum of first 2 terms)

$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}.$

$S_4 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

$S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots + \frac{1}{2^n}$

$\boxed{S_n = \sum_{j=1}^{n} \frac{1}{2^j}}$ ($n^{th}$ partial sum of the series)

Nov 9-6:53 PM

Why do we care about series?

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$\{a_1, a_2, a_3, \dots, a_n, \dots\} \leftarrow$ infinite sequence

$\sum_{k=1}^{\infty} a_k \leftarrow$ series

Partial Sums : $S_1 = a_1$ ; $S_2 = a_1 + a_2$, $S_3 = a_1 + a_2 + a_3$,

$S_n = \sum_{k=1}^{n} a_k$ (sum of first n terms)

Nov 9-6:59 PM