Partial Sums of Arithmetic series

$\{a_1, a_1+d, a_1+2d, a_1+3d, \dots\}$ ← arithmetic sequence.

$$\boxed{S_n = \frac{n(a_1+a_n)}{2}}$$

$a_1 + d + a_1 + (n-2)d = a_1 + \boxed{a_1+(n-1)d} \to a_n$

Why?

$$S_n = a_1 + (a_1+d) + (a_1+2d) + \dots + (a_1+(n-1)d)$$
$$+ \quad S_n = a_n + \underbrace{(a_1+(n-2)d)}_{a_{n-1}} + \underbrace{(a_1+(n-3)d)}_{a_{n-2}} + \dots + a_1$$

$$2S_n = (a_1+a_n) + (a_1+a_n) + (a_1+a_n) + \dots + (a_1+a_n)$$
$$2S_n = (a_1+a_n)\, n \text{ n terms} \qquad \boxed{S_n = \frac{(a_1+a_n)n}{2}}$$

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E.g. $\{5, 8, 11, 14, \dots\}$ ← arithmetic sequence

$3 \quad 3 \quad 3$ common difference $= 3$

$5 + 8 + 11 + 14 + 17 + \dots + \boxed{32} \overset{?}{=} S_{10}$

$a_1 \qquad a_n$

$\boxed{a_n = a_1 + (n-1)\cdot 3} \to a_n = a_1 + (n-1)d$

$32 = 5 + (n-1)\cdot 3$

$27 = (n-1)\cdot 3$

$\boxed{n = 10}$

$$S_{10} = \frac{(a_1+a_{10})\cdot 10}{2} = \frac{(5+32)\cdot 10}{2} = \boxed{185}$$

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(b) $\boxed{20} + 15 + 10 + \dots + \boxed{(-50)}$ $\quad d = -5$

$a_1 \qquad a_n$

$$= \frac{(20+(-50))\cdot 15}{2} = \boxed{-225}.$$

$a_n = a_1 + (n-1)\cdot(-5)$

$-50 = 20 + (n-1)\cdot(-5)$

$-70 = (n-1)(-5)$

$14 = n-1$

$n = 15$

(c) $\displaystyle\sum_{k=1}^{13}(3k-8)$; $a_1 = -5$; $a_2 = -2$; $a_3 = 1$; $a_4 = 4$

$d = 3$

$$= \frac{(-5+31)\cdot 13}{2} = 169. \quad a_1 = -5;\ a_{13} = 31.$$

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$$S_n = \frac{(a_1+a_n)\cdot n}{2} \quad \text{(arithmetic sequence)}$$

Ex (1) Find the sum of the arithmetic series:

$1.4 + 1.6 + 1.8 + \dots + 3.4 = \frac{(1.4+3.4)\cdot 11}{2} = 26.4$

(2) $13 + 21 + \dots + 69 = \frac{(13+69)\cdot 8}{2} = 328$

(3) $\displaystyle\sum_{k=1}^{10}(5-6k) = \frac{(-1+(-55))\cdot 10}{2} = -280$

Partial Sum of a geometric sequence

$$S_n = a_1 + a_1 r + a_1 r^2 + \dots + a_1 r^{n-1}$$

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Formula: $\boxed{S_n = a_1 \cdot \frac{1-r^n}{1-r}}$

Why is this true?

$$S_n = a_1 + \cancel{a_1 r} + \cancel{a_1 r^2} + \cancel{a_1 r^3} + \dots + \cancel{a_1 r^{n-1}}$$
$$r S_n = \cancel{a_1 r} + \cancel{a_1 r^2} + \cancel{a_1 r^3} + \dots + \cancel{a_1 r^{n-1}} + a_1 r^n$$

$r S_n - S_n = a_1 r^n - a_1$

$S_n(r-1) = a_1(r^n - 1)$

$$S_n = a_1 \cdot \frac{r^n - 1}{r-1} = a_1 \frac{1-r^n}{1-r}$$

E.g. $8 + (-4) + 2 + \dots$ $\qquad S_{11} = ?$

$r \leftarrow \boxed{-\frac{1}{2}} \quad -\frac{1}{2}$

$$S_{11} = 8 \cdot \frac{1-\left(-\frac{1}{2}\right)^{11}}{1-\left(-\frac{1}{2}\right)}$$

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Ex: (1) Find S_{20} for the series:

$1000 + 500 + 250 + \dots$

(2) Find $\displaystyle\sum_{k=1}^{8} 3\cdot 2^k$ using the formula for partial sums of geo. series.

(1) $S_{20} = 1000 \cdot \frac{1-\left(\frac{1}{2}\right)^{20}}{1-\frac{1}{2}} = 1999.9981$

(2) $S_8 = 6 \cdot \frac{1-2^8}{1-2} = 1530$

Sum of an infinite geometric series

$$\sum_{n=1}^{\infty} 2^n = \infty$$

geometric series with $r = 2$

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