$$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$$

geometric series with $\boxed{r = \frac{1}{2}}$

In general, the sum of a infinite geometric series will be equal to a finite number if $-1 < r < 1$

$$\sum_{n=1}^{\infty} a_1 r^{n-1} = \frac{a_1}{1-r} \quad \text{if } -1 < r < 1$$

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E.g. ① $\displaystyle\sum_{n=1}^{\infty} 27 \cdot \left(\frac{1}{3}\right)^{n-1} = \frac{27}{1-\frac{1}{3}} = \frac{27}{\frac{2}{3}} = \frac{81}{2}$

$r = \frac{1}{3}$

② $\underbrace{2 + \frac{2}{3}}_{\frac{1}{3}} \underbrace{+ \frac{2}{9}}_{\frac{1}{3}} + \cdots = \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}} = \boxed{3}$

③ $\displaystyle\sum_{k=1}^{\infty} \left(-\frac{3}{8}\right)^k = \frac{-\frac{3}{8}}{1+\frac{3}{8}} = \frac{-\frac{3}{8}}{\frac{11}{8}} = -\frac{3}{11}$

$r = -\frac{3}{8}$

④ $\displaystyle\sum_{k=1}^{\infty} \frac{1}{9}\left(\boxed{\frac{4}{3}}\right)^k = \infty$

$r > 1$

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$$0.\overline{3} = 0.3333\ldots = \frac{1}{3}$$

$$0.\overline{659} = 0.659659659\ldots$$

$$\boxed{0.\overline{659}} = 0.659 + 0.000659 + 0.000000659 + \cdots$$

$$= \frac{659}{1000} + \frac{659}{1000^2} + \frac{659}{1000^3} + \frac{659}{1000^4} + \cdots$$

$\frac{1}{1000}$ $\frac{1}{1000}$ $\frac{1}{1000}$

$$= \frac{\frac{659}{1000}}{1 - \frac{1}{1000}} = \frac{\frac{659}{1000}}{\frac{999}{1000}} = \boxed{\frac{659}{999}}$$

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