11.6 Binomial Theorem

Binomial Coefficient

$$C(n,r) = \frac{n!}{r!\,(n-r)!}$$

Alternative notation: $\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$

(read as n choose r)

$\binom{n}{r}$ is called a binomial coefficient.

E.g. $\binom{5}{3} = \frac{5!}{3!\,2!} = \frac{5\times4\times\cancel{3!}}{\cancel{3!}\,2!} = \frac{20}{2} = 10$

$\binom{9}{2} = \frac{9!}{2!\,7!} = \frac{9\times8\times\cancel{7!}}{2!\,\cancel{7!}} = \frac{72}{2} = 36.$

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$\binom{9}{7} = \frac{9!}{7!\,2!} = \frac{9\times8\times\cancel{7!}}{\cancel{7!}\,2!} = \frac{72}{2} = 36.$

Note: $\binom{9}{2} = \binom{9}{7}$

In general, $\binom{n}{r} = \binom{n}{n-r}$

Binomial Theorem

Binomial: polynomial with 2 terms

Binomial Theorem tells us how to expand the power of a binomial

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$$\boxed{(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k}$$

$$= \binom{n}{0}x^{\boxed{n}} + \boxed{\binom{n}{1}}x^{\boxed{n-1}}y + \boxed{\binom{n}{2}}x^{\boxed{n-2}}y^2 + \cdots + \boxed{\binom{n}{n-2}}x^2y^{n-2} + \boxed{\binom{n}{n-1}}xy^{n-1} + \binom{n}{n}y^n$$

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$$(x+y)^3 = \sum_{k=0}^{3} \binom{3}{k} x^{3-k} y^k$$

$$= \boxed{\binom{3}{0}}x^3 + \binom{3}{1}x^2y + \binom{3}{2}xy^2 + \boxed{\binom{3}{3}}y^3$$

$$= x^3 + \frac{3!}{1!\,2!}x^2y + \frac{3!}{2!\,1!}xy^2 + y^3$$

$$(x+y)^3 = 1x^3 + \boxed{3}x^2y + \boxed{3}xy^2 + 1y^3$$

E.g. Use binomial theorem to expand

$(x+y)^4 =$

$(x+y)^5 =$

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$(x+y)^4 = x^{\boxed{4}} + \boxed{4}x^{\boxed{3}}y + 6x^{\boxed{2}}y^2 + 4xy^3 + y^4$

$(x+y)^5 =$

$\binom{4}{1} = \frac{4!}{1!\,3!} = \frac{4\times\cancel{3!}}{\cancel{3!}} = 4$

$\binom{4}{2} = \frac{4!}{2!\,2!} = \frac{4\times3\times\cancel{2!}}{2!\,\cancel{2!}} = \frac{12}{2} = 6$

$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Pascal Triangle: Easier way to calculate the binomial coefficient.

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1 1

1 2 1

1 3 3 1

1 4 6 4 1

$n=5 \to$ 1 5 10 10 5 1

$n=6 \to$ 1 6 15 20 15 6 1

$(x+y)^2 = 1x^2 + \boxed{2}xy + 1y^2$

$(x+y)^3 = 1x^3 + \boxed{3}x^2y + \boxed{3}xy^2 + y^3$

$(x+y)^4 = 1x^4 + \boxed{4}x^3y + \boxed{6}x^2y^2 + \boxed{4}xy^3 + 1y^4$

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