$$\lim_{x\to 5}\frac{\frac{5-x}{5x}}{x-5}=\lim_{x\to 5}\frac{\frac{-\cancel{(x-5)}}{5x}}{\cancel{x-5}}=\lim_{x\to 5}\frac{-1}{5x}=\boxed{-\frac{1}{25}}$$

Ex:

$$\lim_{x\to -5}\frac{\frac{1}{5}+\frac{1}{x}}{10+2x}=\lim_{x\to -5}\frac{\frac{\cancel{5+x}}{5x}}{2\cancel{(x+5)}}=\lim_{x\to -5}\frac{1}{10x}$$

$$=\frac{-1}{50}$$

Evaluating limits containing a square root.

$$(A-B)(A+B)=A^2-B^2$$

$$\lim_{h\to 0}\frac{(\sqrt{16-h}-4)}{h}\cdot\frac{(\sqrt{16-h}+4)}{(\sqrt{16-h}+4)}$$

$$\lim_{h\to 0}\frac{(\cancel{16}-h)-\cancel{16}}{h\cdot(\sqrt{16-h}+4)}=\frac{-\cancel{h}}{\cancel{h}(\sqrt{16-h}+4)}$$

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$$\lim_{h\to 0}\frac{-1}{\sqrt{16-h}+4}=\boxed{\frac{-1}{8}}$$

Ex: Find limits:

① $\displaystyle\lim_{x\to 0}\frac{x}{\sqrt{1+2x}-1}$

② $\displaystyle\lim_{x\to 9}\frac{x^2-81}{3-\sqrt{x}}$

① $$\lim_{x\to 0}\frac{x}{\sqrt{1+2x}-1}\cdot\frac{\sqrt{1+2x}+1}{\sqrt{1+2x}+1}$$

$$\lim_{x\to 0}\frac{\cancel{x}\cdot(\sqrt{1+2x}+1)}{\cancel{1}+2\cancel{x}\cancel{-1}}=\lim_{x\to 0}\frac{\sqrt{1+2x}+1}{2}=\frac{2}{2}=\boxed{1}$$

② $$\lim_{x\to 9}\frac{(x-9)(x+9)}{3-\sqrt{x}}\cdot\frac{3+\sqrt{x}}{3+\sqrt{x}}=\frac{-\cancel{(x-9)}(x+9)\cdot(3+\sqrt{x})}{\cancel{9-x}}$$

$$\lim_{x\to 9}[-(x+9)(3+\sqrt{x})]=\boxed{-108}$$

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Limits of Piecewise functions

$$f(x)=\begin{cases}\boxed{2x^2+2x+1} & \text{if } x\le 0\\ \boxed{x-3} & \text{if } x>0\end{cases}$$

change to $2x+1$

$$\lim_{x\to 0^+}f(x)=\lim_{x\to 0}(x-3)=-3$$

$$\lim_{x\to 0^-}f(x)=\lim_{x\to 0}(2x^2+2x+1)=1$$

$$\lim_{x\to 0}f(x)\ \text{DNE}.$$

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Basic Properties of Limits

$$\lim_{x\to a}f(x)=L\ ;\quad \lim_{x\to a}g(x)=K$$

$$\lim_{x\to a}(f(x)\pm g(x))=L\pm K$$

$$\lim_{x\to a}(f(x)\cdot g(x))=L\cdot K$$

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{L}{K}\quad\text{provided } K\neq 0.$$

If c is a constant; then $\displaystyle\lim_{x\to a}(c\cdot f(x))=c\cdot L$

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