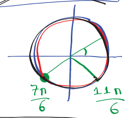
7.5 Trigonometric Equations

(1) Linear equations in sine and cosine

E.g. $2\sin x + 1 = 0$; ~~$0 \le x < 2\pi$~~

$2\sin x = -1$

$\sin x = -\frac{1}{2}$



$x = \frac{7\pi}{6}$; $x = \frac{11\pi}{6}$.

$\boxed{x = \frac{7\pi}{6} + k\cdot 2\pi \; ; \; x = \frac{11\pi}{6} + k\cdot 2\pi}$;

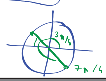
k is any integer

E.g. $2(\tan x + 3) = 5 + \tan x$; $\boxed{0 \le x < 2\pi}$.

$2\tan x + 6 - \tan x - 5 = 0$

$\tan x + 1 = 0$; $\tan x = -1$.

$\boxed{x = \frac{3\pi}{4} ; x = \frac{7\pi}{4}}$



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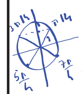
(2) Equations that involve a single trig function.

$2\sin^2\theta - 1 = 0$; $\boxed{0 \le \theta < 2\pi}$.

$2\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$



$\sin\theta = \frac{\sqrt{2}}{2}$ or $\sin\theta = -\frac{\sqrt{2}}{2}$.

$\boxed{\theta = \frac{\pi}{4}, \frac{3\pi}{4}}$; $\boxed{\theta = \frac{5\pi}{4} ; \frac{7\pi}{4}}$

E.g. $\csc^2 x - 4 = 0$; $0 \le x < 2\pi$.

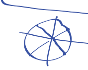
$\csc^2 x = 4$

$\csc x = 2$ or $\csc x = -2$

$\frac{1}{\sin x} = 2$; $\frac{1}{\sin x} = -2$

$\sin x = \frac{1}{2}$; $\sin x = -\frac{1}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6} ; \frac{7\pi}{6}, \frac{11\pi}{6}}$



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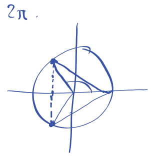
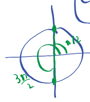
E.g. $2\cos^2\theta + \cos\theta = 0$; $0 \le \theta < 2\pi$.

$\cos\theta(2\cos\theta + 1) = 0$

Either $\cos\theta = 0$ or $2\cos\theta + 1 = 0$

$\boxed{\theta = \frac{\pi}{2}, \frac{3\pi}{2}}$; $\cos\theta = -\frac{1}{2}$; $\boxed{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}}$





(3) Equations quadratic in form

$6 = -2\cdot(-3)$

E.g. $2\sin^2\theta - 5\sin\theta + 3 = 0$

$2\sin^2\theta - 2\sin\theta - 3\sin\theta + 3 = 0$

$2\sin\theta(\sin\theta - 1) - 3(\sin\theta - 1) = 0$

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$(2\sin\theta - 3)(\sin\theta - 1) = 0$

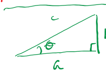
$2\sin\theta = 3$ or $\sin\theta = 1$

~~$\sin\theta = \frac{3}{2}$~~ or $\sin\theta = 1$

$\boxed{\theta = \frac{\pi}{2}}$.

E.g. $\boxed{\sin^2\theta = 2\cos\theta + 2}$; $0 \le \theta < 2\pi$.

Trig Identity: $\boxed{\sin^2\theta + \cos^2\theta = 1}$



Pythagorean Theorem

$a^2 + b^2 = c^2$

$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$

$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$

$1 - \cos^2\theta = 2\cos\theta + 2$

$0 = \cos^2\theta + 2\cos\theta + 1$

$0 = (\cos\theta + 1)(\cos\theta + 1)$

$\cos\theta = -1$; $\boxed{\theta = \pi}$.

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$\boxed{\sin^2\theta + \cos^2\theta = 1}$ $\boxed{1 + \cot^2\theta = \csc^2\theta}$

$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$

$\boxed{\tan^2\theta + 1 = \sec^2\theta}$

$\cos(\alpha + \beta) \ne \cos\alpha + \cos\beta$

$\boxed{\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$ (cosine of a sum)

$\boxed{\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$ (cosine of a difference)

$\boxed{\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha}$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

(Double angle formula)

$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$
$= 1 - 2\sin^2\alpha$
$= 2\cos^2\alpha - 1$

$\sin^2\alpha = (1 - \cos^2\alpha)$

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$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$.

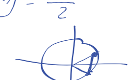
(Double angle formula)

$\sin(2\alpha) = 2\sin\alpha\cos\alpha$

E.g. (1) $\boxed{\cos(x)\cos(2x) + \sin(x)\sin(2x)} = \frac{\sqrt{3}}{2}$

(2) $9\cos(2\theta) = 9\cos^2\theta - 4$

(3) $\boxed{6\sin(2t) + 9\sin(t) = 0}$



(1) $\cos(x - 2x) = \frac{\sqrt{3}}{2}$; $\cos(-x) = \frac{\sqrt{3}}{2}$

$\cos(x) = \frac{\sqrt{3}}{2}$; $\boxed{x = \frac{\pi}{6} ; \frac{5\pi}{6}}$.

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