

8.4. Graphs of equations in polar coordinates.

$$\begin{aligned}x &= r \cos \theta \\y &= r \sin \theta \\ \tan \theta &= \frac{y}{x} \\r^2 &= x^2 + y^2 \quad r = \sqrt{x^2 + y^2}\end{aligned}$$

Symmetry Test for Polar Equations

$$\begin{aligned}\sin(-\theta) &= -\sin \theta \quad \cos(-\theta) = \cos \theta \\(r, \theta) &\rightarrow (-r, -\theta) \quad (r, \theta) \rightarrow (r, \pi)\end{aligned}$$

$\boxed{\theta = \frac{\pi}{2}}$   $(r, \theta)$  and  $(-r, -\theta)$  are symmetric w.r.t.  $\theta = \frac{\pi}{2}$

Oct 10-6:00 PM

E.g.:  $r = 2 \sin \theta$

Replace  $r$  by  $-r$ ,  $\theta$  by  $-\theta$ .

$$\begin{aligned}-r &= 2 \sin(-\theta) \\-r &= 2 \cdot (-\sin \theta) = -2 \sin \theta \\r &= 2 \sin \theta\end{aligned}$$

1<sup>st</sup> Symmetry Test. If we replace  $(r, \theta)$  by  $(-r, -\theta)$  in a polar equation and still get the same equation (after simplification), then the graph of the equation is symmetric w.r.t. the line  $\theta = \frac{\pi}{2}$  (y-axis).

Oct 10-6:10 PM

2<sup>nd</sup> Symmetry Test. If we replace  $(r, \theta)$  by  $(r, -\theta)$  in a polar equation and still get the same equation after simplification, then the equation is symmetric with respect to the line  $\theta = 0$  (x-axis).

E.g.:  $r = 2 \cos \theta$

$$\begin{aligned}r &= 2 \cos(-\theta) = 2 \cos \theta\end{aligned}$$

Oct 10-6:14 PM

3<sup>rd</sup> test of symmetry:

If we replace  $(r, \theta)$  by  $(-r, \theta)$  in a polar equation and we still get the same equation, then the graph of the equation is symmetric with respect to the pole (origin).

E.g.:  $r^2 = \sin(2\theta)$

$$\begin{aligned}(-r)^2 &= \sin(2\theta) \\r^2 &= \sin(2\theta) \\(r, \theta) &\rightarrow (-r, \theta) \quad \text{same equation} \\(r, \theta) &\rightarrow (r, \theta) \quad \text{same equation} \\(r, \theta) &\rightarrow (-r, -\theta) \quad \text{same equation}\end{aligned}$$

Oct 10-6:20 PM

Polar Equations of Circles

$r = a \cos \theta$   $a > 0$   $a < 0$

$r = a \sin \theta$   $a > 0$   $a < 0$

E.g.:  $r = 4 \cos(\theta)$   $\rightarrow$  symmetric w.r.t. x-axis.

Key points: Set  $r = 0$   $4 \cos 0 = 0$   $\cos 0 = 0$   $\theta = \frac{\pi}{2} + k\pi$

$(0, \frac{\pi}{2})$  belongs to the graph

Oct 10-6:26 PM

Max. max of  $\cos \theta$  is 1

$$\begin{aligned}\cos \theta &= 1, \theta = 0 \\r &= 4 \cos(0) = 4\end{aligned}$$

$(1, 0)$  belongs to the graph

$\theta$	$0$	$\frac{\pi}{2}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$r = 4 \cos \theta$	4	2 $\sqrt{3}$	2 $\sqrt{2}$	2	0	-2	-2 $\sqrt{3}$	-2 $\sqrt{2}$	-4

Only need  $\theta \in [0, \pi]$  b/c of symmetry

Oct 10-6:33 PM