8.5. Polar form of a complex number.

$z = 4i$

$z = 2 + 4i$; $\boxed{z = \boxed{-5} + \boxed{\tfrac{1}{2}}i}$

* A complex number is a number of the form $z = \boxed{x} + \boxed{y}i$ where x and y are real #s. And i is called the imaginary unit, $\boxed{i^2 = -1}$

x is called the real part of z.
y is called the imaginary part of z

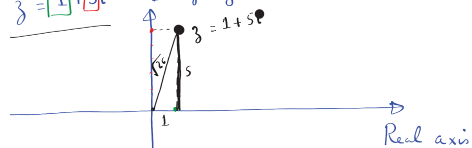
* $\boxed{(\boxed{2+3i})(4+5i)} = 8 + 10i + 12i + 15i^2$
$= 8 - 15 + 22i$
$= \boxed{\boxed{-7} + \boxed{22}i}$

$(2+3i)^2$ $(2+3i)^3$ $\boxed{(2+3i)^4}$

Oct 10-7:03 PM

* Plot a complex # in the plane.

$z = \boxed{1} + \boxed{5}i$



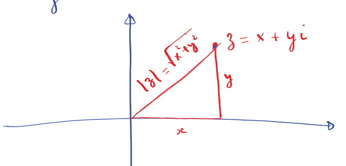
* Absolute value of a complex number.

$z = x + yi$.

The absolute value (modulus) of z is denoted $|z|$ and is given by the formula: $|z| = \sqrt{x^2 + y^2}$

Oct 10-7:25 PM

Geometrically.



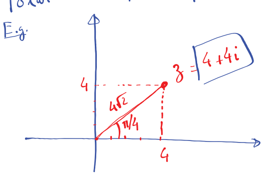
$z = 3 - 4i$; $|z| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$

$z = 12 - 5i$; $|z| = \sqrt{12^2 + (-5)^2} = \sqrt{144 + 25}$
$= \sqrt{169} = \boxed{13}$

Oct 10-7:28 PM

Polar Form of a complex number.

E.g.



$\boxed{\left(4\sqrt{2}, \frac{\pi}{4}\right)}$

polar coordinates for the point $(4,4)$

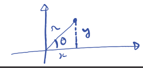
$4 = \boxed{4\sqrt{2}} \cos \boxed{\frac{\pi}{4}}$

$4 = \boxed{4\sqrt{2}} \sin \boxed{\frac{\pi}{4}}$

$z = 4 + 4i$

$z = 4\sqrt{2} \cos \frac{\pi}{4} + \left(4\sqrt{2} \sin \frac{\pi}{4}\right) i = \boxed{4\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)}$

Polar form of $z = x + iy$ $\longrightarrow$ $z = r\cos\theta + i r\sin\theta$



$r = \sqrt{x^2 + y^2}$; $x = r\cos\theta$

$\tan\theta = \frac{y}{x}$; $y = r\sin\theta$

$\boxed{z = r(\cos\theta + i\sin\theta)}$

modulus argument.

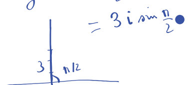
Oct 10-7:30 PM

E.g. Convert the complex # to polar form.

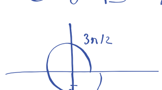
(a) $\boxed{z = 3i}$ (b) $z = -2i$

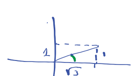
(c) $z = -4 + 4i$ (d) $\boxed{z = \sqrt{3} + i}$

$z = 3\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$
$= 3i \sin \frac{\pi}{2}$



(b) $z = \boxed{2}\left(\cos \boxed{\frac{3\pi}{2}} + i \sin\left(\boxed{\frac{3\pi}{2}}\right)\right)$





(c) $z = \boxed{4\sqrt{2}}\left(\cos \boxed{\frac{3\pi}{4}} + i \sin \boxed{\frac{3\pi}{4}}\right)$

$\sqrt{(-4)^2 + 4^2}$

(d) ~~$z = 2$~~

$x = \sqrt{3}$; $y = 1$

$\sqrt{x^2 + y^2} = 2$

$\boxed{\tan\theta = \frac{1}{\sqrt{3}}}$

$\theta = \frac{\pi}{6}$

$z = \boxed{2}\left(\cos \boxed{\frac{\pi}{6}} + i \sin \frac{\pi}{6}\right)$

Oct 10-7:36 PM

Convert from polar form to rectangular form

$z = 12\left(\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right)\right)$

$z = 12 \cos\left(\frac{\pi}{6}\right) + i \; 12 \sin\left(\frac{\pi}{6}\right)$

$z = 12 \cdot \frac{\sqrt{3}}{2} + i \cdot 12 \cdot \frac{1}{2}$

$z = \boxed{6\sqrt{3}} + i \cdot \boxed{6}$

Real part: $6\sqrt{3}$

Imaginary part: 6

Product of complex numbers in polar form.

Oct 10-7:51 PM