$z_1 = \boxed{2}\left(\cos(\boxed{25^\circ}) + i\sin(25^\circ)\right)$

$z_2 = \boxed{3}\left(\cos(\boxed{35^\circ}) + i\sin(35^\circ)\right)$

$z_1 \cdot z_2 = 6\left(\cos(60^\circ) + i\sin(60^\circ)\right)$

$= 6\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= 3 + i\frac{\sqrt{3}}{2}$

Multiply moduli

Add arguments

$z_1 = 2\left(\cos\left(\frac{3\pi}{5}\right) + i\sin\left(\frac{3\pi}{5}\right)\right)$

$z_2 = 6\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$

$z_1 \cdot z_2 = 12\left(\cos\left(\frac{3\pi}{5} + \frac{\pi}{4}\right) + i\sin\left(\frac{3\pi}{5} + \frac{\pi}{4}\right)\right)$

$= 12 \cdot \left(\cos\left(\frac{17\pi}{20}\right) + i\sin\left(\frac{17\pi}{20}\right)\right)$

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General formula for multiplying 2 complex numbers in polar form.

If $z_1 = R_1\left(\cos\theta_1 + i\sin\theta_1\right)$

$z_2 = R_2\left(\cos\theta_2 + i\sin\theta_2\right)$

then $z_1 \cdot z_2 = \boxed{R_1 \cdot R_2}\left(\cos\left(\boxed{\theta_1 + \theta_2}\right) + i\sin\left(\boxed{\theta_1 + \theta_2}\right)\right)$

Notation: short hand notation for $z_1 = R_1\left(\boxed{\cos\theta_1 + i\sin\theta_1}\right)$

is $z_1 = R_1\,\boxed{\text{cis}}(\theta_1)$

$$\boxed{\left(R_1\,\text{cis}(\theta_1)\right) \cdot \left(R_2\,\text{cis}(\theta_2)\right) = R_1R_2\,\text{cis}(\theta_1 + \theta_2)}$$

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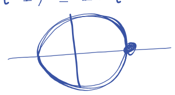
$\boxed{z = 4\,\text{cis}\left(\frac{\pi}{4}\right)} = 4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \left(2\sqrt{2} + i2\sqrt{2}\right)$

$z^2 = 16\,\text{cis}\left(\frac{\pi}{2}\right) = 16\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$= 16(i \cdot 1) = 16i$

$z^3 = 64\,\text{cis}\left(\frac{3\pi}{4}\right)$



$z^4 = 4^4\,\text{cis}\left(\frac{4\pi}{4}\right)$

$z^{2016} = 4^{2016}\,\text{cis}\left(2016 \cdot \frac{\pi}{4}\right)$

$= 4^{2016} \cdot \left(\cos\left(2016 \cdot \frac{\pi}{4}\right) + i\sin\left(2016 \cdot \frac{\pi}{4}\right)\right)$

$= 4^{2016} \cdot \left(\cos(504\pi) + i\sin(504\pi)\right)$

$= 4^{2016} \cdot (1 + i \cdot 0) = \boxed{4^{2016}}$

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Why is this true?

$z_1 = R_1\left(\cos\theta_1 + i\sin\theta_1\right);\ z_2 = R_2\left(\cos\theta_2 + i\sin\theta_2\right)$

$z_1 \cdot z_2 = \boxed{R_1R_2\left(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right)}$

$z_1 \cdot z_2 = R_1R_2\left(\cos\theta_1 + i\sin\theta_1\right)\left(\cos\theta_2 + i\sin\theta_2\right)$

$= R_1R_2\left(\boxed{\cos\theta_1\cos\theta_2} + \cos\theta_1\,\boxed{i}\sin\theta_2 + \cos\theta_2\,\boxed{i}\sin\theta_1 + \underbrace{\boxed{i^2}}_{-1}\sin\theta_1\sin\theta_2\right)$

$= R_1R_2\left(\boxed{\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2} + i\left(\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1\right)\right)$

$= R_1R_2\left(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right)$

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Quotient of Complex #s in polar form

If $z_1 = R_1\,\text{cis}(\theta_1)$

$z_2 = R_2\,\text{cis}(\theta_2)$

then $\boxed{\frac{z_1}{z_2} = \frac{R_1}{R_2}\,\text{cis}(\theta_1 - \theta_2)}$

(Divide moduli, Subtract arguments)

E.g. $z_1 = 2\left(\cos(213^\circ) + i\sin(213^\circ)\right)$

$z_2 = 4\left(\cos(33^\circ) + i\sin(33^\circ)\right)$

$\frac{z_1}{z_2} = \frac{1}{2}\,\text{cis}(180^\circ)$

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Find powers of complex #s in polar form.

$z = R\,\text{cis}(\theta)$

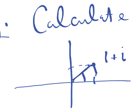
$z^2 = R^2\,\text{cis}(2\theta)$

$z^3 = R^3\,\text{cis}(3\theta)$

In general, if $n$ is any positive integer, then

$\boxed{z^n = R^n\,\text{cis}(n\theta)}$ De Moivre's Theorem

E.g. Calculate $(1+i)^5$ using De Moivre's Theorem.




$1 + i = \sqrt{2} \cdot \text{cis}\left(\frac{\pi}{4}\right)$

$(1+i)^5 = (\sqrt{2})^5 \cdot \text{cis}\left(\frac{5\pi}{4}\right)$

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