8.5

Imaginary Axis

(x, y)

r

θ

Real Axis

$z = x + yi$

$r = \sqrt{x^2 + y^2}$

$x = r\cos\theta$; $y = r\sin\theta$

$z = r\cos\theta + (r\sin\theta)i$

$z = r(\cos\theta + i\sin\theta)$

$z = r\,\mathrm{cis}(\theta)$

$z_1 = r_1\,\mathrm{cis}(\theta_1)$; $z_2 = r_2\,\mathrm{cis}(\theta_2)$

$z_1 z_2 = r_1 r_2\,\mathrm{cis}(\theta_1 + \theta_2)$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\mathrm{cis}(\theta_1 - \theta_2)$

$z = r\,\mathrm{cis}(\theta)$

$z^n = r^n\,\mathrm{cis}(n\theta)$

Oct 12-6:03 PM

8.6. Parametric Equations

y

y shadow

y

150 ft/s

30°

x - Shadow

x

x, y: variables

x and y are changing with time t

$x = f(t)$; $y = g(t)$ → pair of parametric equations.

For our example: $x = (150\cos(30°))\cdot t$;

Oct 12-6:09 PM

$y = -\frac{1}{2}(32)t^2 + (150\sin(30°))t$

$$\begin{cases} x = (150\cos(30°))t \\ y = -16t^2 + (150\sin(30°))t \end{cases}$$

This is an example of a pair of parametric equations.

Ex: Ana is a long distance runner.

Average velocity is 8 mph → parameter.

$x = 8t$ ← distance Ana travels as a function of time.

Bill: 2 hours after Ana leaves the house. Bill leaves in his car and follow the same route. Average velocity 40 mph

$y = 40(t - 2)$

Oct 12-6:15 PM

(1) Plot parametric equations by making a table of values.

E.g. $\begin{cases} x = t \\ y = t^2 - 1 \end{cases}$

Plot the curve parameterized by this pair of parametric equations

t	x	y
-4	-4	15
-3	-3	8
-2	-2	3
-1	-1	0
0	0	-1
1	1	0
2	2	3
3	3	8
4	4	15

y

x

Oct 12-6:20 PM

Ex: Plot the curve parameterized by the parametric equations: $\begin{cases} x = t - 3 \\ y = 2t + 4 \end{cases}$ $-1 \le t \le 2$

$t = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2$

t	x	y
-1	-4	2
$-\frac{1}{2}$		
0		
$\frac{1}{2}$		
1		
2		

line segment

y

$(-1, 8)$

$(-4, 2)$

x

Slope $= \frac{8-2}{-1-(-4)} = \frac{6}{3} = 2$

$y = 2x + b$

$2 = 2\cdot(-4) + b$

$2 = -8 + b$

$b = 10$

$y = 2x + 10$

Write an equation of the line passing through 2 points.

Oct 12-6:26 PM

(x_1, y_1) and (x_2, y_2)

(x_2, y_2)

$y_2 - y_1$

(x_1, y_1)

$x_2 - x_2$

* Slope $= \frac{y_2 - y_1}{x_2 - x_1} = m$

* $y = mx + b$

Plug (x_1, y_1) into the equation.

$y_1 = mx_1 + b$; $b = y_1 - mx_1$.

Point - Slope Formula

$y - y_1 = m(x - x_1)$

$(-4, 2)$; $(-1, 8)$. Slope $= 2$

$y - 2 = 2\cdot(x - (-4))$

$y - 2 = 2(x + 4)$

$y - 2 = 2x + 8$

$y = 2x + 10$

Oct 12-6:36 PM