8.8 - Vectors.

Motivation describe quantities that have both direction and magnitude.

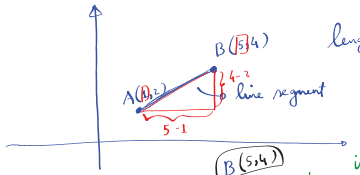
Fly airplane — how fast; direction

force — how strong (magnitude); direction

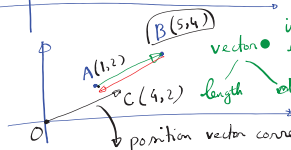
A vector — magnitude; direction.

Oct 24-6:08 PM

* Geometric representation of a vector



length $= \sqrt{(5-1)^2 + (4-2)^2} = \sqrt{16+4} = \sqrt{20}$



initial point A
endpoint B
vector
length
direction
position vector corresponding the vector from A to B

Notation: the vector with initial point A and endpoint B is denoted by $\overrightarrow{AB}$. $\vec{v} = \overrightarrow{AB}$

Oct 24-6:12 PM

Position vector (of $\overrightarrow{AB}$) is the vector which has the same length and direction as $\overrightarrow{AB}$ but starts at the origin $(0,0)$.

If $\overrightarrow{AB}$ has $A = (1,2)$ and $B(5,4)$, then its position vector is $\overrightarrow{OC}$ with $O = (0,0)$ and $C(4,2)$

In general, if the vector $\overrightarrow{AB}$ has $A = (x_1, y_2)$ and $B = (x_2, y_2)$, then the position vector corresponding to $\overrightarrow{AB}$ is the vector with initial point at $(0,0)$ and endpoint at $(x_2 - x_1, y_2 - y_1)$

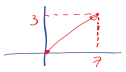
Oct 24-6:21 PM

E.g. $\vec{v}$ — initial point $(-3,2)$; endpoint is $(4,5)$

The position vector $\vec{u}$ corresponding to $\vec{v}$

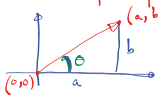
$\vec{u}$ — initial point $(0,0)$; endpoint $(7,3)$

$\vec{u} = (7,3)$



Magnitude and Direction of a position vector.

$\vec{v} = (a,b)$



Magnitude $= \sqrt{a^2+b^2}$

Notation: $|\vec{v}| = \sqrt{a^2+b^2}$

Direction is given angle θ when $\tan\theta = \frac{b}{a}$

Oct 24-6:24 PM

E.g $\vec{v}$ — initial point $P(-8,1)$; endpoint $Q(-2,-5)$

Q: Find the position vector and magnitude and direction of $\vec{v}$.

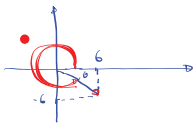
$\vec{u} = (6,-6)$

$|\vec{u}| = \sqrt{72} = 6\sqrt{2}$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\tan\theta = -1$

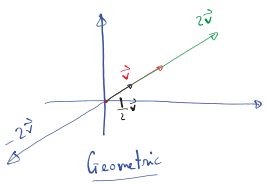
$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$



Oct 24-6:30 PM

Vector Operations

① Scalar multiplication



Geometric

$2\vec{v}$

$\frac{1}{2}\vec{v}$

$-2\vec{v}$

Algebraically

$\vec{v} = (a,b)$

$2\vec{v} = (2a, 2b)$

$\frac{1}{2}\vec{v} = (\frac{1}{2}a, \frac{1}{2}b)$

$-3\vec{v} = (-3a, -3b)$.

In general, if $\vec{v} = (a,b)$, then $k\vec{v} = (ka, kb)$

k scalar

Oct 24-6:34 PM