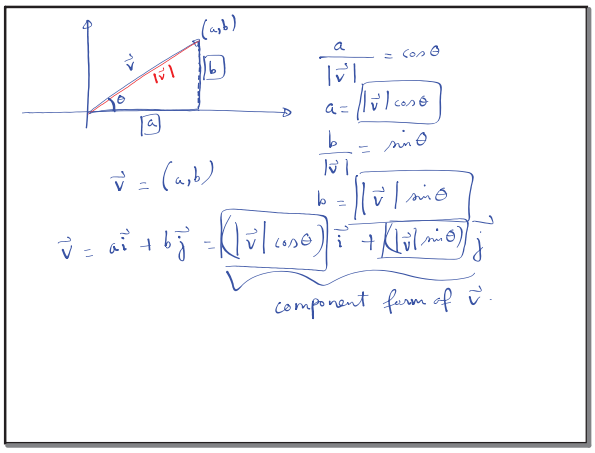


$\frac{a}{|\vec{v}|} = \cos\theta$

$a = |\vec{v}|\cos\theta$

$\frac{b}{|\vec{v}|} = \sin\theta$

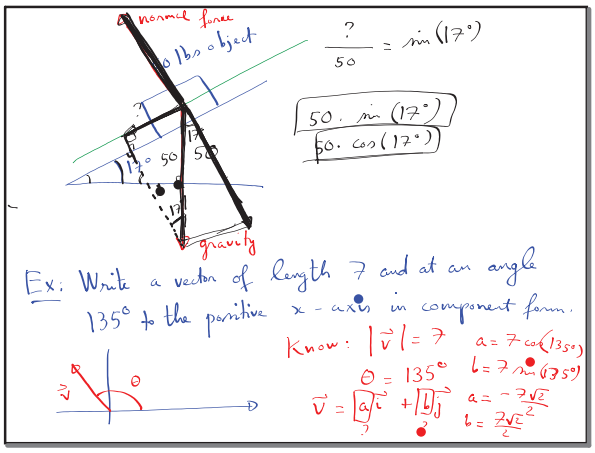
$b = |\vec{v}|\sin\theta$

$\vec{v} = (a,b)$

$\vec{v} = a\vec{i} + b\vec{j} = (|\vec{v}|\cos\theta)\vec{i} + (|\vec{v}|\sin\theta)\vec{j}$

component form of $\vec{v}$.

Oct 24-7:24 PM



$\frac{?}{50} = \sin(17°)$

$50 \cdot \sin(17°)$

$50 \cdot \cos(17°)$

Ex: Write a vector of length 7 and at an angle 135° to the positive x-axis in component form.

Know: $|\vec{v}| = 7$, $\theta = 135°$

$a = 7\cos(135°)$, $b = 7\sin(135°)$

$a = -\frac{7\sqrt{2}}{2}$, $b = \frac{7\sqrt{2}}{2}$

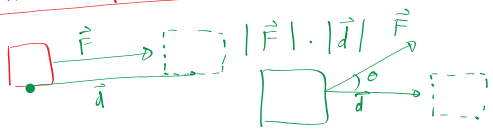
$\vec{v} = a\vec{i} + b\vec{j}$

Oct 24-7:28 PM

$\vec{v} = -\frac{7\sqrt{2}}{2}\vec{i} + \frac{7\sqrt{2}}{2}\vec{j}$

$\vec{v} = \left(-\frac{7\sqrt{2}}{2}, \frac{7\sqrt{2}}{2}\right)$

Dot Product of Two Vectors



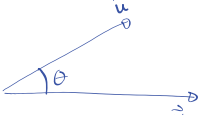
$|\vec{F}| \cdot |\vec{d}|$

Oct 24-7:39 PM

If $\vec{u}$ and $\vec{v}$ are two vectors then the dot product of $\vec{u}$ and $\vec{v}$ is a number defined as

$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cos\theta$

θ is the angle between $\vec{u}$ and $\vec{v}$



If $\vec{u} = (a,b)$ and $\vec{v} = (c,d)$, it turns out that

$\vec{u} \cdot \vec{v} = ac + bd$

Oct 24-7:41 PM

Eg: $\vec{u} = (5,12)$, $\vec{v} = (-3,4)$

$\vec{u} \cdot \vec{v} = 5(-3) + 12 \cdot 4 = -15 + 48 = 33$

Find the angle θ between $\vec{u}$ and $\vec{v}$?

$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cdot \cos\theta$

$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|} = \frac{33}{13 \cdot 5} = \frac{33}{65}$

$\theta = \cos^{-1}\left(\frac{33}{65}\right) \approx 59.5°$

Oct 24-7:44 PM

The angle θ between vectors $\vec{u}$ and $\vec{v}$ can be calculated by:

$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|}$

$= (5,2)$

Ex: Let $\vec{u} = 5\vec{i} + 2\vec{j}$; $\vec{v} = 3\vec{i} + 7\vec{j} = (3,7)$

(a) Find $\vec{u} \cdot \vec{v}$

$\vec{u} \cdot \vec{v} = 5 \cdot 3 + 2 \cdot 7$

$= 15 + 14$

$= 29$

(b) Find the angle θ between $\vec{u}$ and $\vec{v}$.

$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|} = \frac{29}{\sqrt{29}\sqrt{58}}$

$= \frac{\sqrt{29}}{\sqrt{58}} = \sqrt{\frac{29}{58}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\theta = 45°$

Oct 24-7:47 PM