3.5 Derivatives of basic trigonometric functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

E.g. $f(x) = \sin x \cdot \cos x$. Find $f'(x)$?

Sep 26-4:02 PM

$f'(x) = (\sin x)' \cdot \cos x + \sin x \cdot (\cos x)'$

$= \cos x \cdot \cos x + \sin x \cdot (-\sin x)$

$f'(x) = \cos^2 x - \sin^2 x \quad (= \cos(2x))$

E.g. $\frac{d}{dx}(5x^2 \sin x) = 5\frac{d}{dx}(x^2 \sin x)$

$= 5 \cdot (2x \cdot \sin x + x^2 \cdot \cos x)$

Sep 26-4:05 PM

$\frac{d}{dx}\left(\frac{x}{\cos x}\right) = \frac{1 \cdot \cos x - x \cdot (-\sin x)}{(\cos x)^2}$

$= \frac{\cos x + x \sin x}{\cos^2 x}$

E.g. A particle moves along a line. Its position at time t is given by the formula $s(t) = t\sqrt{3} + 2\cos t, \ 0 \le t \le 2\pi$. At what times is the particle at rest?

$v(t) = s'(t) = \sqrt{3} - 2\sin t = 0$

$2\sin t = \sqrt{3}; \ \sin t = \frac{\sqrt{3}}{2}$

$t = \frac{\pi}{3}$ or $t = \frac{2\pi}{3}$.

Sep 26-4:09 PM

E.g. position $s(t) = 2\sin t$.

Find $v\left(\frac{5\pi}{6}\right)$ and $a\left(\frac{5\pi}{6}\right)$

Compare these values and decide whether the particle is speeding up or slowing down at $t = \frac{5\pi}{6}$.

$v(t) = s'(t) = 2\cos t$. $v\left(\frac{5\pi}{6}\right) = 2\cos\left(\frac{5\pi}{6}\right) = 2 \cdot \left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

$a(t) = v'(t) = -2\sin t$

$a\left(\frac{5\pi}{6}\right) = -2\sin\left(\frac{5\pi}{6}\right) = -2 \cdot \frac{1}{2} = -1$

$v\left(\frac{5\pi}{6}\right)$ and $a\left(\frac{5\pi}{6}\right)$ have the same sign.

The particle is speeding up at $t = \frac{5\pi}{6}$.

Sep 26-4:14 PM

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

Sep 26-4:20 PM

$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \cdot \cos x - (-\sin x) \cdot \sin x}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \left(\frac{1}{\cos x}\right)^2$

$= \sec^2 x$

$\frac{d}{dx}(\tan x) = \sec^2 x.$

$\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$

$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$\frac{d}{dx}(\cot x) = \frac{-1}{\sin^2 x} = -\left(\frac{1}{\sin x}\right)^2 = -\csc^2 x$

Sep 26-4:21 PM