

$$\begin{aligned}\frac{d}{dx}(\sec x) &= \frac{d}{dx}\left(\frac{1}{\cos x}\right) \\ &= \frac{0 \cdot \cos x - (-\sin x) \cdot 1}{\cos^2 x} \\ &= \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \cdot \sec x \\ \frac{d}{dx}(\sec x) &= \tan x \cdot \sec x \\ \boxed{\frac{d}{dx}(\csc x)} &= \frac{d}{dx}\left(\frac{1}{\sin x}\right) = \frac{0 \cdot \sin x - \cos x \cdot 1}{\sin^2 x} \\ &= \frac{-\cos x}{\sin^2 x} = \boxed{-\frac{\cos x}{\sin^2 x}} = \boxed{-\cot x \csc x}\end{aligned}$$

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$$\begin{aligned}\frac{d}{dx}(\sin x) &= \cos x & \frac{d}{dx}(\tan x) &= \sec^2 x \\ \frac{d}{dx}(\cos x) &= -\sin x & \frac{d}{dx}(\cot x) &= -\csc^2 x \\ \frac{d}{dx}(\sec x) &= \sec x \tan x & \frac{d}{dx}(\csc x) &= -\csc x \cot x.\end{aligned}$$

E.g. let $f(x) = \csc x + x \tan x$ Find $f'(x)$.

$$\begin{aligned}f'(x) &= -\csc x \cot x + 1 \cdot \tan x + x \sec^2 x \\ &= -\csc x \cot x + \tan x + x \sec^2 x.\end{aligned}$$

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$f(x) = \cot x$ Find the equation of the tangent line to the graph of f at $x = \frac{\pi}{4}$.

$$\begin{aligned}f'(x) &= -\csc^2 x \\ f'\left(\frac{\pi}{4}\right) &= -\csc^2\left(\frac{\pi}{4}\right) = -\frac{1}{\sin^2\left(\frac{\pi}{4}\right)} = -\frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = -\frac{1}{\frac{1}{2}} = -2.\end{aligned}$$

Slope = -2.

Point $\left(\frac{\pi}{4}, 1\right)$

Equation of the tangent line: $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

$$\boxed{y = -2x + \frac{\pi}{2} + 1}$$

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Higher-order derivatives of $y = \sin x$ and $y = \cos x$.

$$\begin{aligned}y &= \sin x \\ \frac{dy}{dx} &= \cos x \\ \frac{d^2 y}{dx^2} &= -\sin x \\ \frac{d^{2016} y}{dx^{2016}} &= \sin x; \quad \frac{d^{2017} y}{dx^{2017}} = \cos x\end{aligned}$$

$$\begin{aligned}\frac{d^3 y}{dx^3} &= -\cos x \\ \frac{d^4 y}{dx^4} &= \sin x \\ \frac{d^{243} y}{dx^{243}} &= -\cos x\end{aligned}$$

In general, $\frac{d^n y}{dx^n}$ where $y = \sin x$. Look at remainder. Divide n by 4.

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$$\begin{aligned}y &= \cos x \\ \frac{dy}{dx} &= -\sin x \\ \frac{d^2 y}{dx^2} &= -\cos x \\ \frac{d^3 y}{dx^3} &= \sin x \\ \frac{d^4 y}{dx^4} &= \cos x \\ \frac{d^n y}{dx^n} &\end{aligned}$$

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