3.7 Derivatives of Inverse Trig functions

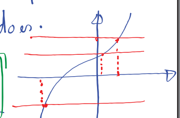
Inverse Function Theorem

Recall: Inverse Functions.

The inverse function of a function is the function that undoes what the original function does.

$\boxed{f(x) = x^3. \quad f^{-1}(x) = \sqrt[3]{x}}$

$f(x) = x+1. \quad f^{-1}(x) = x-1.$



Remark here: f has an inverse only if f is one-to-one.

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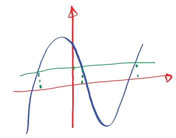

$f(x) = x^3; \quad f^{-1}(x) = \sqrt[3]{x}.$

$f^{-1}(f(x)) = f^{-1}(x^3) = \sqrt[3]{x^3} = x$

$\boxed{f^{-1}(f(x)) = x}$

$f(f^{-1}(x)) = f(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x.$

$f(f^{-1}(x)) = x$

$$\boxed{\begin{aligned} f^{-1}(f(x)) &= x \\ f(f^{-1}(x)) &= x \end{aligned}}$$

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Inverse Function Theorem

Suppose $f(x)$ is an invertible and differentiable function.

Suppose $y = f^{-1}(x)$. the inverse function of f.

If $\boxed{f'(f^{-1}(x)) \neq 0}$ for some x, then

$$\boxed{(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}}$$

E.g. $f(x) = \dfrac{x+6}{x-2}$; $x > 2$. Find $(f^{-1})'(3)$

$(f^{-1})'(3) = \dfrac{1}{f'(f^{-1}(3))}$

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$(f^{-1})'(3) = \dfrac{1}{f'(\boxed{f^{-1}(3)})}$ (By the I.F.T)

* Derivative of f. $f(x) = \dfrac{x+6}{x-2}$. $f'(x) = \dfrac{1\cdot(x-2) - 1\cdot(x+6)}{(x-2)^2}$.

$f'(x) = \dfrac{\cancel{x}-2 \cancel{-x}-6}{(x-2)^2} = \dfrac{-8}{(x-2)^2}$. $\boxed{f'(x) = \dfrac{-8}{(x-2)^2}}$

* Find $\boxed{f^{-1}(3)}$ If we can find a # x such that $f(x) = 3$, then $f^{-1}(3) = x$.

$\underbrace{\dfrac{x+6}{x-2}}_{f(x)} = 3$; $\quad x+6 = 3x-6$

$12 = 2x$

$\boxed{x = 6}$

$f(6) = 3$

$f^{-1}(3) = \boxed{6}$.

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$(f^{-1})'(3) = \dfrac{1}{f'(6)}$

$f'(x) = \dfrac{-8}{(x-2)^2}$; $f'(6) = \dfrac{-8}{16} = -\dfrac{1}{2}$.

$(f^{-1})'(3) = \dfrac{1}{-\frac{1}{2}} = \boxed{-2}$.

Remark: $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$.

let $g(x) = f^{-1}(x)$

If g is the inverse function of f, then $\boxed{g'(x) = \dfrac{1}{f'(g(x))}}$

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Derivatives of Inverse Trig functions.

Why the IFT is true?

$$\boxed{(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}}$$

$\boxed{f(f^{-1}(x))} = \boxed{x}$

Take the derivatives of both sides w.r.t. x

$\boxed{f'(f^{-1}(x))}\boxed{(f^{-1})'(x)} = 1$

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