3.8 - Implicit Differentiation

(1) Find derivatives implicitly.

(2) Find equations of tangent lines implicitly.

$y =$ formula in x (explicitly)

$y = x^2 + 2x - 3$ $\quad \frac{dy}{dx}$ $\quad x = 1$, $x = 2$ find y

Implicit relation $x^2 + y^2 = 25$ $\quad \frac{dy}{dx} = ?$

$x = 2$, find y $\quad 4 + y^2 = 25$

Oct 5-4:03 PM

$x^2 + y^2 = 25$, $y^2 = 25 - x^2$ $\Rightarrow$ $y = \sqrt{25 - x^2}$, $y = -\sqrt{25 - x^2}$ } inconvenient.

Implicit Differentiation

$x^2 + y^2 = 25$. Find $\frac{dy}{dx}$?

Take the derivative with respect to x of both sides

$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(25)$

$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$

$2x + 2y \cdot \frac{dy}{dx} = 0$

$z = y^2$; $\frac{d}{dx}(z)$

y: a function of x

z — y — x

$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx} = 2y \cdot \frac{dy}{dx}$

Oct 5-4:09 PM

$2x + 2y \cdot \frac{dy}{dx} = 0$

$2y \cdot \frac{dy}{dx} = -2x$

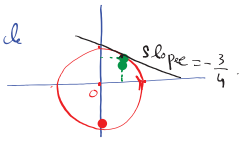
$\frac{dy}{dx} = \frac{-2x}{2y}$

$\frac{dy}{dx} = \frac{-x}{y}$

What is $\frac{dy}{dx}$ when $x = 3$ and $y = 4$

$\frac{dy}{dx} = \frac{-3}{4}$

$x^2 + y^2 = 25$ → equation of a circle



Oct 5-4:15 PM

E.g. $4x^5 + \tan y = y^2 + 5x$. Find $\frac{dy}{dx}$.

$\frac{d}{dx}(4x^5 + \tan y) = \frac{d}{dx}(y^2 + 5x)$

$\frac{d}{dx}(4x^5) + \frac{d}{dx}(\tan y) = \frac{d}{dx}(y^2) + \frac{d}{dx}(5x)$

$20x^4 + \sec^2 y \cdot \frac{dy}{dx} = 2y \cdot \frac{dy}{dx} + 5$

$\sec^2 y \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} = 5 - 20x^4$

$\frac{dy}{dx}(\sec^2 y - 2y) = 5 - 20x^4$

$\frac{dy}{dx} = \frac{5 - 20x^4}{\sec^2 y - 2y}$

Find $\frac{dy}{dx}$ when $x = 0$, $y = 0$; $\left.\frac{dy}{dx}\right|_{x=0,\, y=0} = \frac{5}{1} = 5$

Oct 5-4:19 PM

E.g. $x^3 y + x y^3 = -8$. Find $\frac{dy}{dx}$?

$\frac{d}{dx}(x^3 y) + \frac{d}{dx}(x y^3) = 0$

$3x^2 y + x^3 \frac{dy}{dx} + 1 \cdot y^3 + x \cdot 3y^2 \frac{dy}{dx} = 0$

$\frac{dy}{dx}(x^3 + 3xy^2) = -3x^2 y - y^3$

$\frac{dy}{dx} = \frac{-3x^2 y - y^3}{x^3 + 3xy^2}$

Oct 5-4:24 PM

$y \sin(xy) = y^2 + 2$. Find $\frac{dy}{dx}$.

$\frac{d}{dx}(y \sin(xy)) = \frac{d}{dx}(y^2 + 2)$

$\frac{dy}{dx} \cdot \sin(xy) + y \cdot \frac{d}{dx}(\sin(xy)) = 2y \cdot \frac{dy}{dx}$

$\frac{dy}{dx} \cdot \sin(xy) + y \cdot \cos(xy) \cdot \frac{d}{dx}(xy) = 2y \cdot \frac{dy}{dx}$

$\frac{dy}{dx} \cdot \sin(xy) + y \cdot \cos(xy)(1 \cdot y + x \frac{dy}{dx}) = 2y \frac{dy}{dx}$

$\sin(xy) \cdot \frac{dy}{dx} + y^2 \cos(xy) + xy \cos(xy) \frac{dy}{dx} = 2y \frac{dy}{dx}$

Oct 5-4:28 PM