

3.9 Derivatives of Exp. and Log. functions

- ① Derivatives of Exp. functions
- ② Derivatives of Log. functions
- ③ Logarithmic Differentiation.

Derivatives of Exp. functionsNatural Exp. function: $y = e^x$ If $f(x) = e^x$, then $f'(x) = e^x$.

$\frac{d}{dx}$ Notation: $\boxed{\frac{d}{dx}(e^x) = e^x}$

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$$\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx} \quad (u \text{ is a function of } x)$$

$$(e^{g(x)})' = e^{g(x)} \cdot g'(x).$$

E.g. $\frac{d}{dx}(e^\pi) = 0 \cdot (e^\pi \text{ is a constant})$

$$\frac{d}{dx}(e^{-x}) = e^{-x} \cdot (-1) = -e^{-x}$$

$$\frac{d}{dx}(e^{\sec(x)}) = e^{\sec(x)} \cdot \sec(x) \cdot \tan(x)$$

$$\frac{d}{dx}(x \cdot e^{2x}) = 1 \cdot e^{2x} + x \cdot e^{2x} \cdot 2 = e^{2x} + 2x e^{2x}$$

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Natural Log Function

$$y = \ln x$$

$$\boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}}$$

$$(\ln x)' = \frac{1}{x}$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$$

$$(\ln u)' = \frac{1}{u} \cdot u' = \frac{u'}{u}$$

u a function of x .

$$\boxed{y = \ln x} \quad \text{Find } \frac{dy}{dx}?$$

$$\Leftrightarrow \boxed{e^y = x}$$

$$\frac{d}{dx}(e^y) = \frac{d}{dx}(x)$$

$$e^y \cdot \frac{dy}{dx} = 1$$

$$e^y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

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$$\text{E.g. } \frac{d}{dx}(\ln(\tan x)) = \frac{1}{\tan x} \cdot \cot x = \frac{\cot x}{\tan x} = \cot x$$

Useful properties of the natural log function

$$\ln(uv) = \ln u + \ln v$$

$$\ln\left(\frac{u}{v}\right) = \ln u - \ln v$$

$$\ln(u^a) = a \ln u$$

E.g. $f(x) = \ln((3x+2)^5)$ Find $f'(x)$

$$f(x) = 5 \ln(3x+2); \quad f'(x) = 5 \cdot \frac{1}{3x+2} \cdot 3 = \frac{15}{3x+2}$$

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$$f(x) = \ln \sqrt{5x-7} \quad \text{Find } f'(x)$$

$$f(x) = \ln(5x-7)^{\frac{1}{2}} = \frac{1}{2} \ln(5x-7)$$

$$f'(x) = \frac{1}{2} \cdot \frac{1}{5x-7} \cdot 5$$

$$\boxed{f'(x) = \frac{5}{2(5x-7)}}$$

E.g. $f(x) = \ln \left(\frac{(2x+1)^5}{\sqrt{x^2+1}} \right)$ Find $f'(x) = ?$

$$f(x) = \ln(2x+1)^5 - \ln(\sqrt{x^2+1})$$

$$= 5 \ln(2x+1) - \frac{1}{2} \ln(x^2+1)$$

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$$f'(x) = 5 \cdot \frac{1}{2x+1} \cdot 2 - \frac{1}{2} \cdot \frac{1}{x^2+1} \cdot 2x$$

$$= \frac{10}{2x+1} - \frac{x}{x^2+1}$$

Base $\neq e$

$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$$

$$\frac{d}{dx}(b^x) = b^x \ln b$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x \ln e} = \frac{1}{x}$$

$$\frac{d}{dx}(\log_b u) = \frac{1}{u \ln b} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(b^u) = b^u \cdot \ln b \cdot \frac{du}{dx}$$

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