Eg. $\frac{d}{dx}\left(\log_{10}(x^3+1)\right) = \frac{1}{(x^3+1)\cdot\ln(10)}\cdot 3x^2$

$= \frac{3x^2}{(x^3+1)\cdot\ln(10)}$

$\frac{d}{dx}\left(2^{\cos(3x)}\right) = 2^{\cos(3x)}\cdot\ln(2)\cdot(-\sin(3x))\cdot 3$

$= -3\ln(2)\cdot 2^{\cos(3x)}\cdot\sin(3x)$

$\frac{d}{dx}\left(x^\pi\cdot\pi^x\right) = \frac{d}{dx}(x^\pi)\cdot\pi^x + x^\pi\cdot\frac{d}{dx}(\pi^x)$

$= \pi\cdot x^{\pi-1}\cdot\pi^x + x^\pi\cdot\pi^x\cdot\ln(\pi)$

$= \pi^x\cdot x^{\pi-1}\left(\pi + x\cdot\ln(\pi)\right)$

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Method of Logarithmic Differentiation

$y = x^x$; $\frac{dy}{dx} = ?$

Cannot use power rule (Power rule: $\frac{d}{dx}(x^{\text{fixed \#}})$)

Cannot use the rule for $\frac{d}{dx}(b^x)$

(1) Take ln of both sides: $\ln(y) = \ln x^x$

(2) $\ln(y) = x\ln x$.

(3) Implicit differentiation: $\frac{1}{y}\cdot\frac{dy}{dx} = \frac{d}{dx}(x\ln x)$

$\frac{1}{y}\cdot\frac{dy}{dx} = 1\cdot\ln x + x\cdot\frac{1}{x}$.

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$y\left(\frac{1}{y}\cdot\frac{dy}{dx}\right) = (\ln x + 1)\,y$

$\frac{dy}{dx} = (\ln x + 1)\cdot y$

$\frac{dy}{dx} = (\ln x + 1)\cdot x^x$ ← derivative of x^x.

Eg. $y = (\sin x)^{\cos x}$

$\ln y = \ln \sin x^{\cos x}$; $\ln y = \cos x\cdot\ln(\sin x)$

$\frac{1}{y}\cdot\frac{dy}{dx} = -\sin x\cdot\ln(\sin x) + \cos x\cdot\frac{1}{\sin x}\cdot\cos x$

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$y\left(\frac{1}{y}\cdot\frac{dy}{dx}\right) = \left(-\sin x\ln(\sin x) + \frac{\cos^2 x}{\sin x}\right)y$

$\frac{dy}{dx} = \left(-\sin x\ln(\sin x) + \frac{\cos^2 x}{\sin x}\right)\cdot(\sin x)^{\cos x}$

Method of Logarithmic Differentiation. $g(x)$

Take the derivative of $y = f(x)^{g(x)}$

(1) Take ln of both sides $\ln y = \ln f(x)^{g(x)}$

(2) Property of ln : $\ln y = g(x)\cdot\ln(f(x))$

(3) Take $\frac{d}{dx}$ of both sides:

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$\frac{1}{y}\cdot\frac{dy}{dx} = g'(x)\cdot\ln(f(x)) + g(x)\cdot\frac{1}{f(x)}\cdot f'(x)$

$\frac{1}{y}\cdot\frac{dy}{dx} = g'(x)\ln(f(x)) + \frac{g(x)\cdot f'(x)}{f(x)}$

(4) Multiply both sides by y

$\frac{dy}{dx} = \left(g'(x)\ln(f(x)) + \frac{g(x)\cdot f'(x)}{f(x)}\right)y$

(5) Replace y by $f(x)^{g(x)}$

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